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On the p-restricted edge connectivity of the bipartite Kneser graph $H(n,k)$

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Abstract

Given a simple graph G, a p-restricted edge cut is a subset of edges of G whose removal disconnects G, and such that the number of vertices in each component of the resulting graph is at least p. The p-restricted edge connectivity is denoted by λ_p, which is the minimum cardinality over all p-restricted edge cuts. If a p-restricted edge cut (also called a λ_p-cut) exists, then the graph is called p-restricted edge connected, or, for short, λ_p-connected. Obviously, for any λ_p-cut F, $G-F$ has exactly two components, and each component has at least p vertices. If the deletion of any λ_p-cut results in at least one component containing exactly p vertices in the resulting graph, then the graph is called super-λ_p. In this paper, we examine the p-restricted edge connectivity of the bipartite Kneser graph $H(n,k)$ when $n \geq 3k+1$ and show that the graph is super-λ_p for $p \leq 5$.

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1 Introduction

Given a graph G with vertex set $V(G)$ and edge set $E(G)$, let $F \subset E(G)$. Then F is an edge cut if the resulting graph $G - F$ is disconnected. Fàbrega and Fiol [4] proposed the concept of p-restricted edge connectivity. We call F a *p-restricted edge cut* if each component of the resulting graph $G - F$ has at least p vertices, where p is a positive integer. The minimum cardinality over all p-restricted edge cuts, denoted by λ_p, is the *p-restricted edge connectivity*. If p-restricted edge cuts exist, then the graph is called *p-restricted edge connected*. A graph is called *super-λ_p* (sometimes it is also called *optimal-λ_p*) if the deletion of every minimum p-restricted edge cut will result in a component with exactly p vertices. Clearly, 1-restricted edge connectivity is the edge connectivity of G, and 2-restricted edge connectivity is also known as the *super edge connectivity* of G.

Let A, B be two proper subsets of $V(G)$. We denote by $E[A, B]$ the edges with one end in A and the other end in B. If $B = V(G) \setminus A$, then we denote $E[A, B]$ by $C(A)$. Let F be a p-restricted edge cut. If $|F| = \lambda_p$ then F is called a *λ_p-cut* of G. In this case, the graph $G - F$ contains two components A and B. Let A be the smaller component; then $C(A)$ is the λ_p-cut and A is called a *λ_p-fragment* of G. If A is a λ_p-fragment and $|A| = p$, then A is called trivial and is also known as a *λ_p-atom*. The non-trivial λ_p-fragment with minimum cardinality is called a *λ_p-superatom* of G. It is easy to see that every λ_p-superatom A satisfies $p + 1 \le |A| \le |V(G)|/2$.

The Kneser graph was proposed by Kneser in 1955 [7]. Structural properties of Kneser graphs, such as hamiltonicity, chromatic number and matchings, have been studied extensively. Only recently, a conjecture was made [3] in relation to the super-connectivity of Kneser graphs; some progress was made in [2]. Apparently this is not an easy problem to settle. In this paper we will study the connectivity of a closely related graph, which is the bipartite Kneser graph.

The vertices of the bipartite Kneser graph $H(n, k)$ are all k-subsets and all $(n-k)$-subsets of $[n] = \{1, \ldots, n\}$, such that there is an edge between vertices u and v in $H(n, k)$ if and only if $u \subset v$ or $v \subset u$. So clearly $H(n, k)$ is regular. The degree of $H(n, k)$ is $\binom{n-k}{n-2k} = \binom{n-k}{k}$ and the order of $H(n, k)$ is $2\binom{n}{k}$. A graph is *vertex-transitive* if its automorphism group acts transitively on its vertices. Similarly, a graph is *edge-transitive* if its automorphism group acts transitively on its edges. A graph is *symmetric* if its automorphism group acts transitively on ordered pairs of adjacent vertices. Mirafzal and Zafari [10] showed that $H(n, k)$ are vertex transitive, edge-transitive and symmetric. As $H(n, k)$ are symmetric, it is clear that the connectivity of $H(n, k)$ is $\binom{n-k}{k}$, which is equal to its degree. When $k = 1$, $H(n, 1)$ is a Cayley graph.

Since when $n = 2k$, $H(n, k)$ is a null graph, so in this paper we assume that $n \ge 2k+1$. Clearly, when $n = 2k+1$, the girth of $H(n, k)$ is 6, and when $n \ge 2k+2$, the girth of $H(n, k)$ is 4.

Mütze and Su [14] showed that the bipartite Kneser graph $H(n, k)$ has a hamilton cycle when $k \ge 1$ and $n \ge 2k + 1$. Mirafazal [12] proved that the automorphism group of the bipartite Kneser graph $\mathrm{Aut}(H(n, k)) \cong \mathrm{Sym}([n]) \times Z_2$ when $k \ge 1$

and $n \geq 2k + 1$, where Z_2 is the cyclic group of order 2. Kim, Cheng, Liptak and Li [6] and Mirafazal [11] showed that the bipartite Kneser graph $H(2k + 1, k)$ is a regular hyperstar graph $HS(2(n+1), n+1)$. Jin [5] constructed some 1-factorizations of bipartite Kneser graphs by perpendicular arrays when $k = 2$ and n is an odd prime. Mohammadyari and Darafsheh [13] used the transitivity property of the automorphism group of the bipartite Kneser graph to calculate its Wiener, Szeged and Pl indices.

There are many results in p-restricted edge connectivity. Wang et al. [18] studied some sufficient conditions for super p-restricted edge connectivity of graphs with diameter 2. Yuan et al. [20] proved that a bipartite graph with n vertices is super p-restricted edge connected if $\delta(G) \geq (n + 2p + 3)/4$, where $\delta(G)$ is the minimum degree of G. Yang et al. [21] gave a sufficient condition for an optimal 3-restricted edge connected vertex transitive graph to be a super 3-restricted edge connected graph. Balbuena et al. [1] gave some sufficient conditions for super p-restricted edge connectivity of permutation graphs when $p = 2, 3$. Shang and Zhang [15] presented some degree conditions for any triangle free and bipartite graph to be super 3-restricted edge connected. Wang and Zhao [19] presented some degree conditions for graphs to be super 3-restricted edge connected. Sun et al. [16] proved that a connected vertex transitive graph with degree $d > 5$ and girth $g > 5$ is super p-restricted edge connected for any positive integer p with $p \leq 2g$ or $p \leq 10$ if $d = g = 6$.

The following results are for graphs which are symmetric.

Theorem 1.1 *[8] The only connected regular edge-symmetric graphs which are not super edge-connected are the cycles C_n.*

Since $H(n, k)$ is edge-symmetric, by Theorem 1.1 we know that the edge-connectivity of the bipartite Kneser graph $H(n, k)$ is $\lambda(H(n, k)) = \binom{n-k}{k}$. Furthermore, we know that $H(n, k)$ is optimal super edge connected, or in other words, super-λ_2.

An edge cut F is a *cyclic edge cut* if $G - F$ is disconnected and has at least two components containing cycles. A graph G has a cyclic edge cut if and only if it has at least two disjoint cycles. The *cyclic edge connectivity*, denoted by $\lambda(c)$, is the minimum cardinality of a cyclic edge cut over all cyclic edge cuts. Denote by ζ the minimum cardinality over all edge cuts of shortest cycles. A graph is *cyclically optimal* if $\lambda(c) = \zeta$. A graph is *super cyclically edge connected* if when removing any minimum cyclic edge cut, there is at least one component which is a shortest cycle of the graph.

Theorem 1.2 *[17] Let G be a connected edge-transitive graph with the number of vertices in G being at least 6 and the minimum degree being 4. Then G is cyclically optimal.*

Theorem 1.3 *[16] Let G be a cyclically optimal d-regular graph with $d \geq 3$ and girth g at least 3. Let p be a positive integer satisfying $p < g - \frac{2}{d-2}$. Then G is super-λ_p.*

From the above results, it is clear for $n = 2k+1$ that $H(n,k)$ is cyclically optimal when $k \geq 3$. Therefore $H(n,k)$ is super-λ_p for $p \leq 5$ when $n = 2k+1$ and $k \geq 3$.

In the next section, we will first look at some properties of λ_p-superatom. And in Section 3 we will investigate the p-restricted edge connectivity of $H(n,k)$ when $n > 3k$. In the approach we have employed in this paper, we are looking at the possible size of λ_p superatom. Such an approach will rely on a good upper bound of the maximum number of edges in a λ_p fragment. In general, given a graph with girth g, it is hard to know exactly the maximum number of edges in the graph, and thus it is hard to obtain a bound on the edge cut set; so in this paper, we assume that p is close to the girth g. Further discussion on this can be found in the last section of the paper.

2 The Bound of λ_p-superatom

Given a d-regular graph G with girth g, if $p < g$, clearly we have an upper bound on the cardinality of a p-restricted edge cut of the graph G. Considering a λ_p fragment which is a tree of order p, the cardinality of the edge cut corresponding to the λ_p fragment is $p(d-2)+2$, which is the upper bound of the p-restricted edge cut. In the case that $p = g$, the component could be a cycle of order p, and then the upper bound on the cardinality of the p-restricted edge cut is $p(d-2)$.

Next, we look at the bound on the number of vertices in a λ_p-superatom. Clearly, a λ_p-superatom will contain more than p vertices if it exists. If there are no λ_p-superatoms, then the size of a p-restricted edge cut is determined.

Mantel's theorem stated that:

Theorem 2.1 *If a graph G on n vertices contains no triangle, then it contains at most $\lfloor n^2/4 \rfloor$ edges.*

Then we have the following result.

Theorem 2.2 *Let G be a connected d-regular graph with girth $g = 4$ and $d \geq 2$. Let X be a λ_3-superatom of G. Then the cardinality of X is at least $2d-3$.*

Proof: Let $|X| = x$; then $g = 4 \leq x \leq \frac{V(G)}{2}$. Since X is a connected component with at least three vertices, it follows that X contains at least two edges, which implies that $\lambda_3 \leq 3d-4$.

From Theorem 2.1, we have $|E(G[X])| \leq \lfloor x^2/4 \rfloor$. So then

$$|C(X)| = dx - 2|E(G[X])| \geq dx - 2(x^2/4 - 1) = dx - \frac{x^2}{2} + 2.$$

Since λ_3-superatom satisfies $|C(X)| = \lambda_3$, we have

$$3d - 4 \geq dx - \frac{x^2}{2} + 2,$$

which leads to the following inequality:

$$x^2 - 2dx + 6d - 12 \geq 0, \tag{1}$$

$$\Delta = b^2 - 4ac = (2d)^2 - 4(6d - 12) = 4d^2 - 24d + 48,$$

where a, b, c are the coefficients of x^2, x, and the constant term in inequality (1), respectively.

Clearly, when $d \geq 2$, $\Delta > 0$. From the roots of the quadratic function we know that inequality (1) is true when

$$x \leq d - \sqrt{(d-3)^2 + 3},$$

$$\text{or} \quad x \geq d + \sqrt{(d-3)^2 + 3}.$$

Observe that a λ_3-superatom has to contain at least four vertices, and thus we have $x \geq 2d - 2$. □

Using the same approach, we have the following.

Corollary 2.1 *Let G be a connected d-regular graph with girth $g = 4$ and $d \geq 8$. Let X be a λ_4-superatom of G. Then the cardinality of X is at least $2d - 4$.*

Proof: Let $|X| = x$; then $5 \leq x \leq \frac{V(G)}{2}$ and $\lambda_4 \leq 4d - 8$.

From Theorem 2.1, we have $|E(G[X])| \leq \lfloor x^2/4 \rfloor$, and then we have

$$4d - 8 \geq dx - \frac{x^2}{2} + 2,$$

$$x^2 - 2dx + 8d - 20 \geq 0, \tag{2}$$

$$\Delta = b^2 - 4ac = (2d)^2 - 4(8d - 20) = 4d^2 - 32d + 80,$$

where a, b, c are the coefficients of x^2, x, and the constant term in inequality (2), respectively.

Clearly, when $d \geq 2$ then $\Delta > 0$. From the roots of the quadratic function we know that inequality (1) is true when:

$$x \leq d - 1 - \sqrt{(d-4)^2 + 4},$$

$$\text{or} \quad x \geq d - 1 + \sqrt{(d-4)^2 + 4}.$$

Clearly the λ_4-superatom has to contain at least five vertices, and thus we have $x \geq 2d - 3$. □

Similarly, we can obtain the following result.

Corollary 2.2 *Let G be a connected d-regular graph with girth $g = 4$ and $d \geq 2$. Let X be a λ_5-superatom of G. Then the cardinality of X is at least $2d - 5$.*

Furthermore, using the symmetric property of $H(n, k)$, we can get some more information on the superatoms. We know that:

Theorem 2.3 *[9] Let G be a graph, with X_1 and X_2 subsets of $V(G)$. Then*

$$|C(X_1 \cap X_2)| + |C(X_1 \cup X_2)| \leq |C(X_1)| + |C(X_2)|.$$

Therefore we have the following results.

Lemma 2.1 *Let X_1 and X_2 be two p-restricted fragments of G. If $X_1 \cap X_2$ is connected, then $C(X_1 \cap X_2) \leq \lambda_p$. If $X_1 \cap X_2 = C_1 \cup C_2 \cup \cdots \cup C_t$, where C_i is a set of components, then $C(C_1 \cup C_2 \cup \cdots \cup C_t) \leq \lambda_p$ for $1 \leq i \leq t$.*

Proof: Since λ_p is non-decreasing in p, if there is a component X in G which has less than p vertices, then $C(X) \leq \lambda_p$, and if X has more than p vertices, then $C(X) \geq \lambda_p$.

Suppose that $X_1 \cap X_2$ is connected, and $|C(X_1 \cap X_2)| > \lambda_p$. Clearly $|X_1 \cup X_2| \geq p$, and thus $|C(X_1 \cup X_2)| \geq \lambda_p$. Therefore we have

$$2\lambda_p < |C(X_1 \cap X_2)| + |C(X_1 \cup X_2)| \leq |C(X_1)| + |C(X_2)| \leq 2\lambda_p,$$

which is a contradiction.

If $X_1 \cap X_2$ is a set of disconnected components, it is straightforward to see that $C(C_1 \cup C_2 \cup \cdots \cup C_t) \leq \lambda_p$, following the same line of reasoning. □

Lemma 2.2 *Let X_1 and X_2 be two p-restricted superatoms of G with $X_1 \neq X_2$. If $X_1 \cap X_2$ is connected, then $|X_1 \cap X_2| \leq p$. If $X_1 \cap X_2 = C_1 \cup C_2 \cup \cdots \cup C_t$, where C_i is a set of components, then $|C_i| \leq p$ for $1 \leq i \leq t$.*

Proof: From Lemma 2.1 we know that $C(X_1 \cap X_2) \leq \lambda_p$. Because $X_1 \neq X_2$, we have $|X_1 \cap X_2| \leq X_1$ and $|X_1 \cap X_2| \leq X_2$. If $|X_1 \cap X_2| > p$, this means that $X_1 \cap X_2$ is a smaller p-restricted fragment, a contradiction.

If $X_1 \cap X_2$ is a set of disconnected components, it is straightforward to see that no component contains more than p vertices following the same line of reasoning. □

The above lemma tell us that two superatoms could overlap on at most p vertices.

3 Super-connectivity of $H(n, k)$

Let the two partite sets of $H(n, k)$ be A and B. Let X be a p-restricted edge cut and C_1, C_2 be the components of $H(n, k) - X$. Clearly, each component is a bipartite graph. Let the two partite sets of C_i be A_i and B_i for $i = 1, 2$, respectively. Assume that $|C_1| \leq |C_2|$ and $A_i \leq B_i$. We have the following results.

Lemma 3.1 *Let $n \geq 3k$. Then the distance between any two vertices in the same partite set of $H(n, k)$ is 2.*

Proof: Let $x = \{1, \ldots k\}$ and $z = \{a_1, \ldots, a_k\}$ be two vertices in A of $H(n, k)$ and $N(x) \subset B$. If $y \in N(x)$, then $y = \{1, \ldots, k, *\}$, where $*$ are $n - 2k$ labels in $\{k + 1, \ldots n\}$. Since the vertices in $N(x)$ have $n - 2k > k$ labels from $\{k + 1, \ldots n\}$, there must be a vertex $y' = \{1, \ldots, k, a_1, \ldots a_k, \ldots\} \in N(x)$, and y' is adjacent to z. Thus the distance between x and z is 2. Moreover, we have $N(N(x)) = A$. □

However, when $3k > n > 2k + 1$, such a property does not hold. In this case, the vertices from $A - N(N(x))$ will have at least $n - 2k + 1$ labels from $\{k + 1, \ldots, n\}$ and the number of vertices in $A - N(N(x))$ is not zero, as shown in the following.

$$\begin{aligned}\binom{n-k}{n-2k+1}\binom{n-(n-2k+1)}{k-(n-2k+1)} &= \binom{n-k}{k-1}\binom{2k-1}{n-k}\\ &= \frac{(2k-1)!}{(k-1)!(n-2k+1)!(3k-n+1)!}.\end{aligned}$$

When $n \geq 3k + 1$ we have the following results.

Theorem 3.1 *The bipartite Kneser graph $H(n, k)$ is 3-restricted edge connected and super-3-restricted edge connected if $n \geq 3k + 1$ and $k \geq 7$.*

Proof: Let us assume F is a 3-restricted edge cut of $H(n, k)$. Then the graph $H(n, k) - F$ has two components; let the smaller component be C_1 which is a superatom. Clearly, C_1 is a bipartite graph with partite sets A_1 and B_1. Based on Theorem 2.2, the size of C_1 is at least $2d-2$. Also it is easy to see that $|A_1|-|B_1| \leq 1$, or otherwise $d|A_1| - d|B_1| > 2d - 4$, which is larger than the upper bound of λ_3, a contradiction. This also implies that $|A_1| \leq |A|/2$; recall, A is a partite set of $H(n, k)$.

Let the vertex $x \in A_1$; there are two cases to consider. First, assume that $N(x) \subset B_1$, which also implies that $d < |A|/2$. As we have $N(N(x)) = A$, it follows that there are at least $|A - A_1|$ edges inbetween $N(X)$ and $A - A_1$, which is at least $\binom{n}{k}/2$. Now take an edge connecting $N(x)$ and $A - A_1$; assume that the two end vertices are $a \in A - A_1$ and $b \in N(x) \in B_1$. Assume $b = \{1, 2, \ldots, k, b_1, b_2, \ldots b_k, t\}$, and a is $\{b_1, b_2, \ldots b_k\}$. It is easy to see that t could be any label in $n - \{1, 2, \ldots, k, b_1, b_2, \ldots b_k\}$; in other words, there are up to $n - 2k$ options, which implies that a is adjacent to $n - 2k$ vertices in $N(x)$. Thus we know that between $A - A_1$ and $N(x)$ there are at least $(n-2k)\binom{n}{k}/2$ edges, which is larger than $3d - 4$.

Suppose $N(x) \subset B_1$ is not true. If every vertex of C_1 has more than 2 edges connected to vertices not in C_1, then clearly the edge cut set is more than $4d - 4 > 3d - 4$. Thus there must be a vertex x such that $N(x) \cup B_1 \geq d - 1$. Following the same line of reasoning, take an edge that connects $N(x)$ and $A - A_1$; the end vertex $a \in A - A_1$ is adjacent to $k + 1$ vertices in $N(x)$, of which there is at most one edge which is not in B_1. Thus there are more than $(n-2k-1)\binom{n}{k}/2$ edges, which is larger than $3d - 4$ edges, inbetween C_1 and $H - C_1$, a contradiction.

As there is no superatom, it follows that $H(n, k)$ is super 4-restricted edge connected. □

Following the same proof, it is straightforward to see that $(n-2k-2)\binom{n}{k}/2 > 4d-8$ when $k \geq 10$ and $(n-2k-3)\binom{n}{k}/2 > 4d-8$ when $k \geq 13$. We have the following results.

Corollary 3.1 *The bipartite Kneser graph $H(n,k)$ is 4-restricted edge connected and super-4-restricted edge connected if $n \geq 3k+1$ and $k \geq 10$.*

Corollary 3.2 *The bipartite Kneser graph $H(n,k)$ is 5-restricted edge connected and super-5-restricted edge connected if $n \geq 3k+1$ and $k \geq 13$.*

4 Discussion

As shown in this paper, the bipartite Kneser graph $H(n,k)$ is super-λ_p for $p \leq 5$ and $n \geq 3k+1$. Using the same approach, it is not hard to obtain similar results for $p = 6$ or $p = 7$, which is relatively close to the girth g. When p gets larger, estimating a tight upper bound of the p-restricted edge cut becomes a difficult problem, thus requiring a different approach.

The case $n \leq 3k$ is still open, because the nice property of $N(N(X)) = A$ no longer holds, and the graph is indeed less dense compared to the case where $n \geq 3k+1$.

Also of interest is the structure of the superatoms. As we have shown in this paper that the superatoms might overlap on a number of vertices, knowledge of the symmetric property of the superatoms would greatly help in investigating the connectivity of symmetric graphs.

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