ON GRAPHS SATISFYING A STRONG ADJACENCY PROPERTY

W. Ananchuen and L. Caccetta

School of Mathematics and Statistics
Curtin University of Technology
GPO Box U1987
Perth 6001
Western Australia.

Dedicated to the memory of Alan Rahilly, 1947 – 1992

ABSTRACT:

Let m and n be nonnegative integers and k be a positive integer. A graph G is said to have property $P^*(m,n,k)$ if for any set of $m + n$ distinct vertices of G there are exactly k other vertices, each of which is adjacent to the first m vertices of the set but not adjacent to any of the latter n vertices. The case $n = 0$ is, of course, a generalization of the property in the Friendship Theorem. In this paper we show that, for $m = n = 1$, graphs with this property are the so-called strongly regular graphs with parameters $(\frac{(k+t)^2}{t}+1,\ k+t,\ t-1,\ t)$ for some positive integer t. In particular, we show the existence of such graphs. For $m \geq 1$, $n \geq 1$, and $m + n \geq 3$, we show that, there is no graph having property $P^*(m,n,k)$, for any positive integer k.

1. INTRODUCTION

For our purposes, graphs are finite, loopless and have no multiple edges. For the most part, our notation and terminology follows that of

Bondy and Murty [9]. Thus G is a graph with vertex set V(G), edge set E(G), $\nu(G)$ vertices and $\varepsilon(G)$ edges. However, we denote the complement of G by $\bar{G}$. The subgraph of G induced by a subset X of vertices of G is denoted by G[X].

Let G be a graph with the property that for any two vertices in the graph there is a unique vertex adjacent to both of them. The Friendship Theorem states that in such a graph there must be a vertex which is adjacent to all other vertices. Graphs satisfying this property are called **friendship graphs**. By virtue of the Friendship Theorem, a friendship graph is either a triangle or a union of triangles having precisely one vertex in common. This was first proved by Erdös, Rényi and Sós [19], and later alternate proofs were given by Wilf [30] and Longyear and Parsons [24].

Friendship graphs can be generalized in several ways. These generalizations are typically concerned with specifying either the number of paths between any two vertices or the size of the common neighbour set of any m-subset of vertices. We refer the interested reader to Bondy [8], Delorme and Hahn [18] and the articles cited therein.

Heinrich [22] determined all graphs G of order at least m + 1, $m \geq 3$, with the property that for any m-subset A of V(G) there is a unique vertex u, $u \notin A$, which has exactly two neighbours in A.

Caccetta, Erdös and Vijayan [13] studied graphs G with the property that for any subsets A and B of V(G) with $A \cap B = \phi$ and $|A \cup B| = t$, there is a vertex $u \notin A \cup B$ which is joined to every vertex of A but not joined to any vertex of B. This property has also been considered by Bollobás [7]. Exoo [20] considered graphs in which the size of the sets A and B are specified as m and n respectively.

More specifically, a graph G is said to have property P(m,n,k) if for any set of m + n distinct vertices of G there are at least k other vertices, each of which is adjacent to the first m vertices of the set but not adjacent to any of the latter n vertices. This property has been studied by a number of authors. For example, Ananchuen and Caccetta [2], [3], Blass and Harary [5], Blass, Exoo and Harary [6], Caccetta and Vijayan [14], Caccetta, Vijayan and Wallis [15] and Exoo and Harary [21].

Blass and Harary [5] established, using probabilistic methods, that almost all graphs have property P(n,n,1). From this, it is not too difficult to show that almost all graphs have property P(m,n,k). Since almost all graphs have this property, it is of interest to ask what happens if the conditions are varied. For example, what happens if there are exactly k other vertices, each of which is adjacent to the first m vertices of the set but not adjacent to any of the latter n vertices. We consider this question here.

This problem was mentioned by Alspach, Chen and Heinrich [1]. They also characterized the class of triangle-free graphs with a certain adjacency property.

2. PRELIMINARIES

A graph G is said to have property P*(m,n,k) if for any set of m + n distinct vertices of G there are exactly k other vertices, each of which is adjacent to the first m vertices of the set but not adjacent to any of the latter n vertices. The class of graphs having property P*(m,n,k) is denoted by $\mathcal{G}$*(m,n,k). Observe that if

$G \in \mathcal{G}^*(m,n,k)$, then $\bar{G} \in \mathcal{G}^*(n,m,k)$. The cycle C_5 of length 5 is a member of $\mathcal{G}^*(1,1,1)$. The well-known Petersen graph is a member of $\mathcal{G}^*(1,1,2)$. Also, if G has property $P^*(m,n,k)$ then it has property $P(m,n,k)$.

For a graph G and $x \in V(G)$ we write N_x for the neighbour set (in G) of x and $N_{\bar{x}}$ for the non-neighbour set of x. Further, for $A, B \subseteq V(G)$ we write

$$N_A = \bigcap_{x \in A} N_x \ , \quad N_{\bar{A}} = \bigcap_{x \in A} N_{\bar{x}} \ , \quad \text{and}$$

$$N_{A\bar{B}} = N_A \cap N_{\bar{B}} \ .$$

Thus, for example, for $x,y,z \in V(G)$ we may write

$$N_{xy} = N_x \cap N_y \ , \quad N_{x\bar{y}} = N_x \cap N_{\bar{y}} \ ,$$

$$N_{x\bar{y}\bar{z}} = N_x \cap N_{\bar{y}} \cap N_{\bar{z}} \ .$$

Where appropriate, lower case letters will denote the cardinality of the set defined by the corresponding upper case letters. Thus, for example, $n_{xy} = |N_{xy}|$.

The case $\mathcal{G}^*(m,n,k)$ has been studied when one of m or n is zero. Trivially, the only members of $\mathcal{G}^*(1,0,k)$ and $\mathcal{G}^*(0,1,k)$ are the k-regular and the $(\nu-k-1)$-regular graphs, respectively. Erdös, Rényi and Sós [19] proved that a graph $G \in \mathcal{G}^*(2,0,1)$ if and only if G consists of $\frac{1}{2}(\nu-1)$ triangles joined at one common vertex (ie. G $\cong K_1 \vee (\frac{\nu-1}{2} K_2)$). This result is the well-known Friendship Theorem. For other values of m, n and k there is a connection with the class of the so-called strongly regular graphs, first introduced by Bose [10].

An r-regular graph of order ν is called **strongly regular** with parameters (ν,r,λ,μ) if G has the property that any two adjacent vertices have exactly λ common neighbours and any two non-adjacent vertices have exactly μ common neighbours. The following well-known result (see [23] pp.119-124) provides a necessary condition for a graph to be strongly regular.

Theorem 2.1: Let G be a strongly regular graph with parameters (ν,r,λ,μ). Then the following holds.

(i) $r(r-\lambda-1) = \mu(\nu-r-1)$.

(ii) $\bar{G}$ is a strongly regular graph with parameters $(\nu, \nu-r-1, \nu-2r+\mu-2, \nu-2r+\lambda)$.

(iii) The adjacency matrix A of G has three distinct real eigenvalues r, s_1 and s_2 with respective multiplicities 1, m_1 and m_2, satisfying $m_1 + m_2 = \nu - 1$ and $s_1m_1 + s_2m_2 = -r$. Furthermore, s_1 and s_2 are the zeros of the polynomial $f(x) = x^2 - (\lambda-\mu)x - (r-\mu)$, and both are integral unless G has parameters $(4\mu+1, 2\mu, \mu-1, \mu)$. □

Bose and Shrikhande [11] proved the following result.

Theorem 2.2: For $k \geq 2$, $G \in \mathcal{G}^*(2,0,k)$ if and only if G is a strongly regular graph with parameters $(\frac{r(r-1)}{k}+1,\ r,\ k,\ k)$ for some positive integer r and there exists a positive integer s, such that $r = k + s^2$ and s divides k. □

Some constructions of graphs in the class $\mathcal{G}^*(2,0,k)$ are also given in the above-mentioned paper.

The only other known result is the following due to Carstens and Kruse [17] and Sudolsky' [27].

Theorem 2.3: For $m \geq 3$ and $k \geq 1$, $\mathcal{G}^*(m,0,k) = \{K_{m+k}\}$ □

3. THE CASE m = n = 1

In this section we will establish that the members of $\mathcal{G}^*(1,1,k)$ are strongly regular graphs with parameters $(\frac{(k+t)^2}{t}+1,\ k+t,\ t-1,\ t)$. Further, we present some constructions to demonstrate the existence of graphs in this class. We make use of a particular strongly regular graph, the so-called pseudo-cyclic graph.

A strongly regular graph with parameters (4k+1, 2k, k-1, k) is called **pseudo-cyclic (PC) graph**. We note that the complement of a PC-graph is again a PC graph with the same parameters as the original graph. Observe that the simplest PC-graph is the 5-cycle which gives rise to the parameters (5,2,0,1). The following result (see [28], p. 294) provides a necessary condition for the existence of PC-graphs.

Theorem 3.1: A necessary condition for the existence of a PC-graph of order $\nu = 4k + 1$, $k > 0$ is that ν is the sum of the squares of two integers. □

We now present our main result for this section.

Theorem 3.2: $G \in \mathcal{G}^*(1,1,k)$ if and only if G is a strongly regular graph with parameters $(\frac{(k+t)^2}{t}+1,\ k+t,\ t-1,\ t)$ for some positive integer t.

Proof: Let $G \in \mathcal{G}^*(1,1,k)$ and u, v be any two adjacent vertices of G. Then $n_{u\bar{v}} = n_{\bar{u}v} = k$, and so $\delta(G) \geq k + 1$. Consequently any pair of non-adjacent vertices have at least one common neighbour. Hence G is connected. Now suppose that $n_{uv} = t - 1$ for some positive integer t. Then $n_u = k + t = n_v$. Hence G is $(k+t)$-regular. Therefore for any two non-adjacent vertices u, w of G, $n_{uw} = t$. Consider a vertex x of G, we have

$$n_{u\bar{x}} = k \quad , \quad \text{for any } u \in N_x$$

and

$$n_{x\bar{v}} = k \quad , \quad \text{for any } v \in N_{\bar{x}} .$$

Thus

$$\sum_{u \in N_x} n_{u\bar{x}} + \sum_{v \in N_{\bar{x}}} n_{x\bar{v}} = k(\nu-1).$$

Now, in the left hand side of the above equation, the first sum counts the edges between the sets N_x and $N_{\bar{x}}$ whilst the second sum counts the non-edges between the sets N_x and $N_{\bar{x}}$. Therefore

$$\sum_{u \in N_x} n_{u\bar{x}} + \sum_{v \in N_{\bar{x}}} n_{x\bar{v}} = n_x n_{\bar{x}},$$

and so

$$k(\nu-1) = n_x n_{\bar{x}} = n_x(\nu-n_x-1).$$

Consequently

$$\nu = 1 + \frac{n_x^2}{n_x - k}$$

$$= 1 + \frac{(k + t)^2}{t} \quad , \tag{3.1}$$

since $n_x = k + t$. Therefore G is a strongly regular graph with the required parameters.

The converse follows directly from the definition of strongly regular graphs. This completes the proof of the theorem. □

We now present a number of corollaries to Theorem 3.2.

Corollary 1: Let $G \in \mathcal{G}^*(1,1,k)$. Then $\nu(G) \geq 4k + 1$, with equality possible if and only if G is a PC-graph with parameters $(4k+1, 2k, k-1, k)$.

Proof: The right hand side of equation (3.1) achieves its minimum value of $4k + 1$ when $k = t$. Hence $\nu(G) \geq 4k + 1$. Further, if $k \neq t$ then $\nu(G) > 4k + 1$, thus establishing the corollary. □

Corollary 2: Let $G \in \mathcal{G}^*(1,1,k)$ be a non PC-graph. Then $k = \ell(\ell-1)$ for some integer $\ell > 1$.

Proof: This corollary is easily established by the application of Theorem 2.1 (iii). □

Observe that, if k is a prime number or 1, the right hand side of equation (3.1) is possible only if $t = 1$, k or k^2. For these cases the graphs in the class $\mathcal{G}^*(1,1,k)$ are strongly regular graphs with parameters $((k+1)^2+1,\ k+1,\ 0,\ 1)$, $(4k+1,\ 2k,\ k-1,\ k)$ and $((k+1)^2+1,\ k(k+1),\ k^2-1,\ k^2)$, respectively. Strongly regular graphs with parameters $((k+1)^2+1,\ k+1,\ 0,\ 1)$ exists if and only if strongly regular graphs with parameters $((k+1)^2+1,\ k(k+1),\ k^2-1,\ k^2)$ exists (by Theorem 2.1 (ii)). If G is a strongly regular graph with $\lambda = 0$ and $\mu = 1$, then G has girth 5. So a strongly regular graph with parameters $((k+1)^2+1,\ k+1,\ 0,\ 1)$ is a Moore graph. Further, from a result of Hoffman and Singleton (see [4], Chap. 23), it follows that a strongly regular graph with parameters $((k+1)^2+1,\ k+1,\ 0,\ 1)$ exists only if $k + 1 = 2,\ 3,\ 7$ or (possibly) 57. Only three strongly regular graphs with $\mu = 1$ are known (see [12], p. 39) : the cycle of length 5 with parameters (5,2,0,1); the Petersen graph with parameters (10,3,0,1); and the Hoffman-Singleton graph with parameters (50,7,0,1). The existence of the strongly regular graph with parameters (3250,57,0,1) remains an open question. Using this fact, the following corollaries to Theorem 3.2 are directly obtained.

Corollary 3: $\mathcal{G}^*(1,1,1) = \{C_5\}$. □

Corollary 4: $G \in \mathcal{G}^*(1,1,2)$ if and only if G is the Petersen graph, the complement of the Petersen graph or a PC-graph with parameters (9,4,1,2). □

Corollary 5: If k is an odd prime, then $\mathcal{G}^*(1,1,k) = \{G \mid G$ is a PC-graph with parameters $(4k+1,\ 2k,\ k-1,\ k)\}$. □

Figure 3.1 below shows a PC-graph with parameters (9,4,1,2).

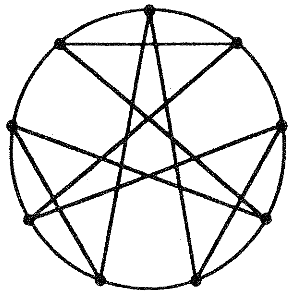

Figure 3.1.

Remark 1. Cameron and Van Lint in [16] (pp.136-137) constructed a class of strongly regular graphs with parameters (243, 22, 1, 2) from ternary Golay codes. These graphs along with their complements are members of $\mathcal{G}^*(1,1,20)$.

Remark 2. A strongly regular graph with parameters $(\frac{(k+2)^2}{2}+1,\ k+2,\ 1,\ 2)$ exists only if k + 2 = 2, 4, 14, 22, 112 or 994 (see [16], p.138). For k + 2 = 2 or 4 the graph is a triangle or a PC - graph of order 9 respectively; an example with k + 2 = 22 is that mentioned in Remark 1. The other cases are undecided.

Remark 3. The Hoffman - Singleton graph, H_{50}, with parameters (50,7,0,1) and its complement are members of $\mathcal{G}^*(1,1,6)$. By using Theorems 2.1 and 3.2, Remark 2 and results from the book [29] (Sections T and U), we get $\mathcal{G}^*(1,1,6) = \{H_{50}, \overline{H}_{50}$, PC-graph with parameters (25,12,5,6), strongly regular graph with parameters (26,10,3,4,) and

(26,15,8,9)}, since there are no strongly regular graphs with parameters (28,9,2,3), (28,18,11,12), (33,8,1,2) and (33,24,17,18).

Remark 4. Let $G \in \mathcal{G}^*(1,1,k)$. If $\nu(G) \le 49$, then we can conclude from Theorems 2.1, 3.1 and 3.2 and its Corollaries and Remark 3 that G must be one of the following graphs.

- a PC-graph of order 5,9,13,17,25,29,37,41,45 or 49.
- the Petersen graph or its complement.
- a strongly regular graph with parameters (26,10,3,4) or (26,15,8,9).

Note that (see [29]) there is only one PC-graph of order 5,9,13 and 17 and there are 15 non-isomorphic PC-graphs of order 25. Further, there are 10 non-isomorphic strongly regular graphs with parameters (26,10,3,4). There are 1504 non-isomorphic PC - graph of order 45 (see [25]).

Remark 5. The well-known Paley graphs [26] provide further examples of graphs in the class $\mathcal{G}^*(1,1,k)$. Let $q = 4k + 1$ be a prime power. The **Paley graph** G_q of order q is the graph whose vertices are elements of the finite field (Galois field) $\mathbb{F}_q$, two vertices are adjacent if and only if their difference is a quadratic residue. G_q is self-complementary. Further, see [7] it is strongly regular with parameters (4k+1, 2k, k-1, k). Thus $G_q \in \mathcal{G}^*(1,1,k)$. Note that the graphs in Figure 3.1 is a Paley graph of order 9.

Remark 6. There exists members of $\mathcal{G}^*(1,1,k)$ that are not PC-graphs. One construction is through partial geometries (Bose [10]). The graph of a partial geometry is obtained by taking the vertices of the graph to correspond to the points of the partial geometry, and taking two vertices to be adjacent if and only if they are incident with the same line of the geometry. A partial geometry with parameters (ℓ,δ,t) (each point is incident to ℓ lines, no two points are incident to more than one line, each line is incident to δ points and if a point P is not incident to a line L, then there are t lines through P intersecting L.) gives rise to a strongly regular graph G with parameters $(\frac{\delta[(\ell-1)(\delta-1) + t]}{t}, \ell(\delta-1), \ell t-1, \ell t)$, provided $\delta = \ell + t$. Thus G $\in \mathcal{G}^*(1,1,\ell(\ell-1))$ for some integer $\ell > 1$.

4. THE CASE $m \geq 1$, $n \geq 1$ and $m + n \geq 3$.

In this section we establish that, there is no graph having property $P^*(m,n,k)$ for $m \geq 1$, $n \geq 1$, $m + n \geq 3$ and $k \geq 1$. We begin with the following simple lemma.

Lemma 4.1: Let $G \in \mathcal{G}^*(m,n,k)$ and let w be any vertex of G. Then for $m \geq 1$, $n \geq 1$ and $k \geq 1$, $G[N_w] \in \mathcal{G}^*(m-1,n,k)$ and $G[N_{\bar{w}}] \in \mathcal{G}^*(m,n-1,k)$. □

Lemma 4.2: If $\mathcal{G}^*(2,1,k) = \phi$, then $\mathcal{G}^*(m,n,k) = \phi$ for any $m \geq 1$, $n \geq 1$ and $m + n \geq 3$.

Proof: Suppose to the contrary that $\mathcal{G}^*(m,n,k) \neq \phi$, and let $m_0 + n_0$ be the smallest value of $m + n \geq 3$ for which $\mathcal{G}^*(m,n,k) \neq \phi$. Then, since $\mathcal{G}^*(1,2,k) = \phi$ when $\mathcal{G}^*(2,1,k) = \phi$, we must have $m_0 + n_0 \geq 4$. Let $G \in \mathcal{G}^*(m_0,n_0,k)$ and w be any vertex of G. By Lemma 4.1, $G[N_w] \in$

$\mathcal{G}^*(m_0-1,n_0,k)$ and $G[N_{\bar{w}}] \in \mathcal{G}^*(m_0,n_0-1,k)$. Since $m_0+n_0 \geq 4$ one of m_0 or n_0 is at least 2 and so, by our assumption, at least one of $\mathcal{G}^*(m_0,n_0-1,k)$ or $\mathcal{G}^*(m_0-1,n_0,k)$ is empty. As w is arbitrary this implies that G is either the complete graph or its complement, which is impossible. Therefore $\mathcal{G}^*(m,n,k) = \phi$. This prove the lemma. □

We now present our main result for this section.

Theorem 4.1: For $m \geq 1$, $n \geq 1$, $m + n \geq 3$ and $k \geq 1$, $\mathcal{G}^*(m,n,k) = \phi$.

Proof: In view of Lemma 4.2, we need only to show that $\mathcal{G}^*(2,1,k) = \phi$. Suppose to the contrary that $\mathcal{G}^*(2,1,k) \neq \phi$. Let $G \in \mathcal{G}^*(2,1,k)$ and let w be any vertex of G. Observe that G cannot be a complete graph and its diameter is 2. Then Lemma 4.1 implies $G[N_w] \in \mathcal{G}^*(1,1,k)$ and $G[N_{\bar{w}}] \in \mathcal{G}^*(2,0,k)$. Hence $G[N_w]$ is a strongly regular with parameters $(\frac{(k+t)^2}{t} + 1,\ k+t,\ t-1,\ t)$ for some positive integer t, and $G[N_{\bar{w}}]$ is a strongly regular with parameters $(\frac{(k+s^2)(k+s^2-1)}{k} + 1,\ k+s^2,\ k,\ k)$ for some positive integer s, by Theorem 3.2 and 2.2 respectively.

We now establish that G is a regular graph. Let $x \in N_w$. Since $G[N_w]$ is (k+t)-regular we have

$$n_{wx} = d_{G[N_w]}(x) = k + t.$$

Now consider x. Using the above argument we can conclude that $G[N_x]$ is (k+t′)-regular for some positive integer t′. Since $w \in N_x$ and $n_{wx} = k + t$ it follows that t′ = t. Consequently, since $G \in \mathcal{G}^*(2,1,k)$, $G[N_y]$ is (k+t)-regular for every $y \in V(G)$. Therefore, since $G[N_w]$ has

$\frac{(k+t)^2}{t}+1$ vertices, G is $(\frac{(k+t)^2}{t}+1)$-regular. Consequently G is a strongly regular graph.

Let $x \in N_w$ and $y \in N_{\bar{w}}$ be non-adjacent vertices of G. Observe that $n_{xw} = \lambda = k + t$ and $n_{xy} = \mu$. Since $G \in \mathcal{G}^*(2,1,k)$, $n_{xw\bar{y}} = k$ and $n_{xy\bar{w}} = k$. Therefore $\mu = n_{xy} = k + t$. Hence G is a strongly regular graph with parameters

$$\nu = \frac{(k+t)^2}{t} + \frac{(k+s^2)(k+s^2-1)}{k} + 3,$$

$$r = \frac{(k+t)^2}{t} + 1,$$

$$\lambda = k + t = \mu,$$

for some positive integers s and t. Further, since $y \in N_{\bar{w}}$ and $G[N_{\bar{w}}]$ is $(k+s^2)$-regular we have

$$n_{wy} = \frac{(k+t^2)}{t} + 1 - (k+s^2).$$

Now since $\mu = k + t = n_{wy}$ we have

$$\frac{k^2}{t} + 1 = s^2. \tag{4.1}$$

Since for any $x \in N_w$ and $y \in N_{\bar{w}}$, $n_{x\bar{w}} = r - \lambda - 1 = \frac{k(k+t)}{t}$ and $n_{yw} = \mu = k + t$, we have

$$(\frac{(k+t)^2}{t}+1)(\frac{k(k+t)}{t}) = (\frac{(k+s^2)(k+s^2-1)}{k}+1)(k+t) \tag{4.2}$$

Equation (4.2) together with (4.1) yields k = 0, a contradiction. This complete the proof of the theorem. □

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