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Even time constraints on the watchman's walk*

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Abstract

In this discrete time process, guards are allowed to traverse edges of a network under the constraint that the neighbourhood of each vertex must be visited within a specified time interval t. Setting $t = 0$ the guards form a dominating set at all times. We show every rooted tree with relatively few leaves has a branch with a certain order and number of pendant vertices. This result is used to establish the validity of a conjecture of Dyer and Milley regarding the maximum number of guards required when t is even.

1 Introduction

Let $G = (V, E)$ be a graph. A *dominating set* of G is a subset of V whose closed neighbourhood is V. A *dominating walk* of G is a walk in G which intersects the closed neighbourhood of each vertex in G. A *minimum closed dominating walk* (MCDW) of G is a shortest walk which is both a closed walk and a dominating walk.

A security firm is asked to indefinitely protect a building, represented by a network, from intruders. Guards standing in any room are able to monitor the room they are in and all adjacent rooms. In an ideal situation, there are enough guards to be placed in a dominating set of the network. In the other extreme, only one guard may be used and the most efficient method of protecting the building would be for the guard to use the shortest dominating walk or shortest MCDW of the network. Hartnell, Rall and Whithead [3] introduced a balanced approach, called the *watchman's walk*, which does not require the rooms to be monitored at all times, but rather gives a maximum time interval, t, during which each room may remain unguarded.

Formally, the watchman's walk process on the graph $G = (V, E)$ can be defined as follows. At time $i = 0$, a set of guards are placed on vertices of a graph (more than

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one guard is allowed on a vertex). During each increment of time, every guard must either stay on their current vertex or move to an adjacent one. A graph is called *t-monitored* and the guards are said to *t-monitor G* if for every interval, $\mathcal{I}$, of positive integers of length $t+1$ and every vertex $v \in V$, there is a guard in $N[v]$, at some time $i \in \mathcal{I}$. Given a reflexive graph G and a set of guards $\mathcal{W}$, a *guarding strategy* is an an assignment of infinite walks on G to each guard in $\mathcal{W}$. The fewest number of guards which can t-monitor G is denoted $W_t(G)$. Observe that $W_0(G) = \gamma(G)$, the cardinality of the smallest dominating set. This version of the problem allows fewer guards to monitor a graph than the original problem involving a minimum dominating set and can be more efficient than spreading out guards on a MCDW. The graph H in Figure 1 has domination number $\gamma(H) = 12$. Allowing any vertex to remain unguarded for up to two time intervals (i.e., $t = 2$) and evenly spreading the guards over the MCDW of H requires nine guards. On the other hand, using two guards on the each of the three closed walks indicated in Figure 1, will 2-monitor H. In particular, it can be shown that $W_2(H) = 6$.

Hartnell, Rall, and Whitehead [3] showed that the MCDW of any tree may be found by deleting the leaves and traversing each edge in the resulting tree exactly twice. As a corollary to this result, Dyer and Milley observed that enough guards, separated by at most $t+1$ steps on a MCDW, can t-monitor a graph. These results are summarized in the following theorem which will be used in Section 2.

Theorem 1.1 *Given a tree, T, with n vertices including L leaves such that $n-L > 1$, the minimum number of guards required to t-monitor the tree for any t with all guards using a MCDW is:*

$$\left\lceil \frac{2(n-L-1)}{t+1} \right\rceil .$$

In particular, $W_t(G) \leq \left\lceil \frac{2(n-L-1)}{t+1} \right\rceil$.

Hartnell and Whitehead [4] described MCDWs for two infinite families of graphs containing large cycles and in [5] found upper and lower bounds on the length of MCDWs in Cartesian product graphs. They further proceeded to characterize graphs meeting the lower bound and construct an infinite family of graphs meeting the upper bound.

In a classic result, Ore [6] showed that a dominating set was at most half of the vertices of a graph. Analogous bounds on $W_t(T)$, where T is a tree, were studied by Davies et al. [1] and established for $t = 1, 2$ and 3. This result was significantly improved by Dyer and Milley [2] who proved the following:

(1) $W_t(T) \leq \left\lfloor \frac{2n+t-3}{t+3} \right\rfloor$ for a tree T when $t > 0$ is odd and this result is best possible and

(2) $W_t(T) \leq \left\lfloor \frac{2n+t-4}{t+2} \right\rfloor$ for a tree T when $t > 0$ is even.

The authors furthered conjectured that the second result could be improved.

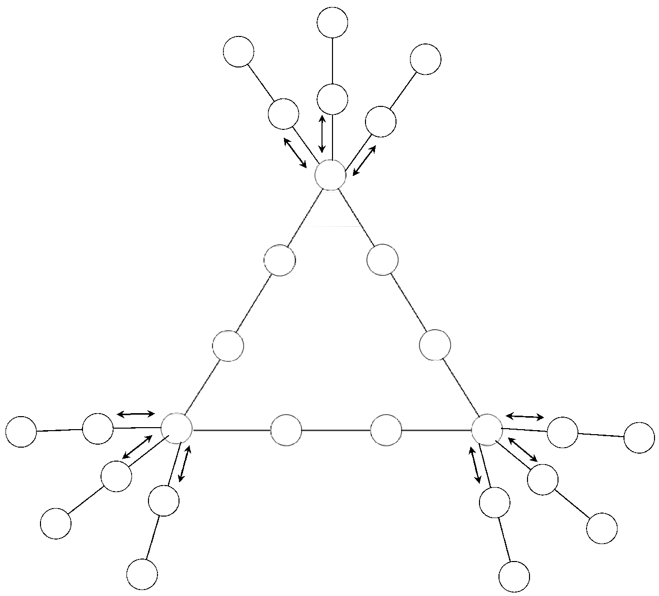

Figure 1: The graph H.

Conjecture 1.2 *[2] If T is a tree of order n and $t > 0$ is an even integer then*

$$W_t(T) \leq \left\lfloor \frac{2n+t-2}{t+3} \right\rfloor.$$

In this paper, we establish a property of rooted trees and use it to prove Conjecture 1.2 is correct. A tree is constructed for each value of n which achieves equality in the bound.

The proof of our main result makes use of the concept of a *branch* of some vertex x in a rooted tree (T, r). The branch is the tree rooted at x, induced by x and all of its descendants.

2 Main Result

In this section we first show that trees with many leaves can be t-monitored with relatively few guards (Lemma 2.1). To prove our main result (Theorem 2.3) we will proceed with induction on the order of the tree. In particular, for trees with relatively few leaves, we remove a rooted subtree containing L leaves and $\lceil L\frac{t+3}{2} \rceil$ vertices (other than the root). This branch is guarded independently. Proposition 2.2 proves the existence of such a branch.

Lemma 2.1 *For a tree T with n vertices and L leaves, if $n \leq Lk+1$, where $k = \frac{t+3}{2}$, then the tree can be t-monitored by $\left\lfloor \frac{2n+t-2}{t+3} \right\rfloor$ guards using a MCDW.*

Proof: If $n \leq (L-1)k+1$, then there exists an integer $L' < L$ such that $(L'-1)k+1 < n \leq L'k+1$.

If the number of vertices is such that $(L'-1)k + 2.5 \leq n \leq L'k+1$, then

$$\begin{aligned} L' &= \frac{L'(t+3)}{t+3} = \frac{(L'-1)(t+3)+t+3}{t+3} \\ &\leq \frac{2n+t-2}{t+3} \leq \frac{L'(t+3)+t}{t+3} \\ &< \frac{(L'+1)(t+3)}{t+3} = L'+1 \end{aligned}$$

so $\left\lfloor \frac{2n+t-2}{t+3} \right\rfloor = L'$. We must show that such a tree can be t-monitored with L' guards. Since T has at most $L'k+1$ vertices, and at least L' leaves, then by Theorem 1.1, there is a dominating walk using

$$\left\lceil \frac{2(n-L-1)}{t+1} \right\rceil \leq \left\lceil \frac{2(L'k+1-L'-1)}{t+1} \right\rceil = \left\lceil \frac{2L'(k-1)}{t+1} \right\rceil = \left\lceil \frac{L'(t+1)}{t+1} \right\rceil = L'$$

guards.

Otherwise, $n = (L'-1)k+2$ or $n = (L'-1)k+1.5$. Then

$$\left\lfloor \frac{2n+t-2}{t+3} \right\rfloor \in \left\{ \left\lfloor \frac{(L'-1)(t+3)+t+2}{t+3} \right\rfloor, \left\lfloor \frac{(L'-1)(t+3)+t+1}{t+3} \right\rfloor \right\} = \{L'-1\}$$

and we must now show that for such a tree $L'-1$ guards are sufficient for a MCDW. Since the number of vertices is at most $(L'-1)k+2$, by Theorem 1.1 we may t-monitor the graph with all guards sharing a dominating walk using

$$\begin{aligned} \left\lceil \frac{2(n-L-1)}{t+1} \right\rceil &\leq \left\lceil \frac{2((L'-1)k+2-L'-1)}{t+1} \right\rceil = \left\lceil \frac{2(L'-1)(k-1)}{t+1} \right\rceil \\ &= \left\lceil \frac{(L'-1)(t+1)}{t+1} \right\rceil = L'-1 \end{aligned}$$

guards. ∎

Given a rooted tree (T, r), let $S(T, r) = \{v \in V(G) : deg(v) = 1 \text{ and } v \neq r\}$.

Proposition 2.2 *Given a rooted tree (T, r), let $L = |S(T, r)|$ and $n = |V(T)|$ be such that $n \geq \lceil Lk \rceil$, where $2k$ is an integer. It follows that for some c, $1 \leq c \leq L$, there exists a branch (T', r') of (T, r) with $c = |S(T', r')|$ and exactly $\lceil ck \rceil$ vertices.*

Proof: We will proceed with induction on L. If $L = 1$, then r is a leaf of a path of at least $\lceil Lk \rceil$ vertices and the result is immediate. Assume the statement is true for all s such that $1 \leq s \leq L - 1$. Now consider a rooted tree (T, r) with $S(T, r) = L$. Let v be the vertex closest to r with at least two children. That is, if r is a leaf, let v be the closest child of r with degree greater than 2 and if r is not a leaf, let $v = r$. Consider the branch of v in (T, r). Call this (T', v) and let $|V(T')| = n'$.

Suppose $n' \leq \lceil Lk \rceil$. There is a vertex y, either v or one of its ancestors, such that the branch of y has exactly size $\lceil Lk \rceil$ vertices with L leaves. The result follows. On the other hand suppose $n' > \lceil Lk \rceil$. Consider any branch, B, in (T', v), of a child v_B of v. Let n_B denote the number of vertices in B and $L_B = |S(B, v_B)|$. If $n_B \geq \lceil L_B k \rceil$ then by the induction hypothesis there exists a branch in B with exactly $\lceil ck \rceil$ vertices and c leaves where $1 \leq c \leq L_B < L$. Since B is a branch of T', then this branch exists in T as well. If $n_B \leq \lceil L_B k \rceil - 1$, consider the rooted tree (T'', v) induced by $V(T') - V(B)$. It follows that $|S(T'', v)| = L - L_B$, and T'' has at least $n' - n_B$ vertices. Hence, since $2k$ is an integer,

$$\begin{aligned} |V(T'')| = n' - n_B &> \lceil Lk \rceil - (\lceil L_B k \rceil - 1) \\ &\geq Lk - (L_B k - 0.5) \\ &= (L - L_B)k + 0.5 \\ &\geq \lceil (L - L_B)k \rceil . \end{aligned}$$

Therefore the number of vertices in T'' is greater than $\lceil (L - L_B)k \rceil$ and by the induction hypothesis, there exists a branch with exactly $\lceil ck \rceil$ vertices and c leaves other than the root where $1 \leq c \leq L - L_B < L$ in T''. This branch does not include v since $n' - n_B > \lceil (L - L_B)k \rceil$. Hence, this branch exists in T. ■

Theorem 2.3 *For an even integer $t > 0$, any tree T on $n \geq 3$ vertices can be t-monitored by $\left\lfloor \frac{2n+t-2}{t+3} \right\rfloor$ guards.*

Proof: We start by verifying the theorem for $3 \leq n \leq \frac{t+6}{2}$. Then

$$1 < \frac{t+4}{t+3} \leq \frac{2n+t-2}{t+3} \leq \frac{(t+6)+t-2}{t+3} = \frac{2t+4}{t+3} < 2$$

so $\left\lfloor \frac{2n+t-2}{t+3} \right\rfloor = 1$. For a tree on $\frac{t+6}{2}$ vertices, there are at least two leaves. There are then $\frac{t}{2} + 3 - 2 - 1 = \frac{t}{2}$ edges that must be traversed twice in order to t-monitor the graph. This takes t time sub-intervals and can be done using one guard.

Assume that any tree on m vertices with $3 \leq m \leq n - 1$ can be t-monitored by $\left\lfloor \frac{2m+t-2}{t+3} \right\rfloor$ guards. Let $k = \frac{t+3}{2}$. Now consider a tree on n vertices, L of which are leaves. If $n \leq Lk + 1$, then by Lemma 2.1, the graph can be guarded using $\left\lfloor \frac{2n+t-2}{t+3} \right\rfloor$ guards. If $n > Lk + 1$, root T at a non-leaf vertex. It is easy to see that $n \geq \lceil Lk \rceil$, and by Proposition 2.2, there is a branch (B, v) in T of size $\lceil ck \rceil$ vertices and $c = |S(B,v)|$, with $1 \le c \le L$. By Theorem 1.1,

$$\begin{aligned} W_t(B) &\le \left\lceil \frac{2(n-L-1)}{t+1} \right\rceil \\ &\le \left\lceil \frac{2}{t+1}\left(\left\lceil \frac{t+3}{2}c \right\rceil - c - 1 \right) \right\rceil \\ &= \left\lceil \frac{2}{t+1}\left(\left\lceil \frac{t+1}{2}c \right\rceil - 1 \right) \right\rceil . \\ &\le \left\lceil \frac{2}{t+1}\left(\frac{t+1}{2}c - 0.5 \right) \right\rceil = c. \end{aligned}$$

Thus B can be t-monitored using c guards. By hypothesis, either $|V(T-B)| \le 2$ or $T-B$ can be t-monitored by $\left\lfloor \frac{2(n-\lceil ck \rceil)+t-2}{t+3} \right\rfloor$ guards. In the former case it can be shown that $n \le Lk+1$, a contradiction. Hence

$$\begin{aligned} W_t(T) &\le \left\lfloor \frac{2(n-\lceil ck \rceil)+t-2}{t+3} \right\rfloor + c \\ &= \left\lfloor \frac{2(n-\lceil c\frac{t+3}{2} \rceil)+t-2+c(t+3)}{t+3} \right\rfloor \\ &\le \left\lfloor \frac{2n+t-2}{t+3} \right\rfloor . \end{aligned}$$

■

Combining this with the previous results, we get the following corollary.

Corollary 2.4 *If G is a connected graph of order n then*

$$W_t(G) \le \begin{cases} \frac{n}{2} & \text{if } t = 0 \\ \frac{2n+t-3}{t+3} & \text{if } t \text{ is odd} \\ \frac{2n+t-2}{t+3} & \text{otherwise.} \end{cases}$$

Proof: The first two bounds have been previously established. In every connected graph there exists a spanning tree, T. Since, by Theorem 2.3, this tree can be t-monitored by $W_t(T)$ guards, G can also be t-monitored by this number of guards by only using the edges in G that are included in T. ■

It was established in [2] this result is best possible for t odd. We now show this result is best possible when t is even.

Let $\mathcal{G}_t$ be the family of graphs as follows. Given an arbitrary tree, $T = (V, E)$, attach two $(\frac{t}{2}+1)$-arms (a k-arm is a path with k vertices, attached to the rest of the graph at a leaf) to each vertex of T except possibly one vertex, say u of T. In the case two $(\frac{t}{2}+1)$-arms are not attached to u one may attach one arm of length $\frac{t}{2}+1$ and one arm of length less than $\frac{t}{2}+1$, or one arm of length at most $\frac{t}{2}+1$ or nothing. We wish to show that for any graph $G \in \mathcal{G}_t$ the number of guards stated in Theorem 2.3 are necessary.

Theorem 2.5 *For any even $t > 0$ and any graph $G \in \mathcal{G}_t$ with $n \geq 3$ vertices, $W_t(G) = \lfloor \frac{2n+t-2}{t+3} \rfloor$.*

Proof: We prove the stronger result that for any even $t > 0$ and any graph $G \in \mathcal{G}_t$, $\lfloor \frac{2n+t-2}{t+3} \rfloor$ watchmen are required so that each leaf of G is monitored every $t+1$ units of time. Let t be the smallest positive integer such that there exists a graph in $\mathcal{G}_t$ which is a counter example. In [1] the authors showed that $t > 2$.

The structure of G allows one to verify the result holds for $3 \leq n \leq t+7$. Assume the result is true for $3 \leq n \leq m$ $(m \geq t+7)$ and let G be any element of $\mathcal{G}_t$ with $m+1$ vertices. Suppose $\mathcal{S}$ is a guarding strategy which t-monitors the leaves of G using fewer than $\lfloor \frac{2(m+1)+t-2}{t+3} \rfloor$ guards and note that we may assume no guard ever visits a leaf.

Recall that G was formed by taking a tree T and attaching two $(\frac{t}{2}+1)$-arms at all but possibly one vertex. Let x be an arbitrary vertex in the copy of T with at least one arm of length greater than one attached. Further suppose s is a stem on (one of) the arm(s) attached to x in the process of forming G. We wish only to ensure the leaf of G is watched every $t+1$ units of time. Hence we may assume that if a guard moves from x to a vertex on the arm containing s, the guard will proceed directly to s. Similarly we may assume if a guard leaves s, it will proceed directly to x. In addition we can assume that there will never be more than one guard on this arm. Certainly, we can assume two guards will not start on the same arm as one could start on the corresponding stem and the other on the corresponding vertex of T or both could start on the corresponding vertex of T. For larger times, if there was a time where two watchmen w_1 and w_2 were on this arm, the guard which was first to arrive on s (say w_1) could have remained on s until w_2 was supposed to arrive and taken over the remainder of w_2's route (or kept w_1's route if w_1 remained on s throughout w_2's entire stay on s). During this process, w_2 could have remained on x until one of w_1 or w_2 was supposed to arrive and taken over the remainder of this route.

Consider a leaf v of T at which two $(\frac{t}{2}+1)$-arms were attached to form G. Let u be the neighbour of v in T. Let the subgraph consisting of v and the two $(\frac{t}{2}+1)$-arms attached at v be called F. Let s_1 and s_2 be the stems of F. Call the first two guards to visit s_1 and s_2, w_1 and w_2, respectively. (It may be the case that $w_1 = w_2$. In this case, suppose without loss of generality that w_1 visited s_1 first (at time $t = 0$ or $t = 1$). Label the first watchman, other than w_1 to visit s_1 as w_2.)

We claim that one can assume that w_1 and w_2 are the only guards which visit s_1 and s_2. Suppose not. That is, suppose there is a smallest time t^* when a guard $w \notin \{w_1, w_2\}$ visits one of these stems, say s_1. To prove the claim we show one can always increase the value of t^* by adjusting the strategy of the guards.

There is a largest time i smaller than t^* when s_1 was visited by a watchman. The definition of t^* implies this watchman was either w_1 or w_2. Without loss of generality it is w_1. At time $i+\frac{t}{2}$, w_1 is on v. As s_1 must be visited every $t+1$ time units and no other watchman will enter the arm containing s_1, w moves from v to the arm containing s_1 at time $t' \in \{i+\frac{t}{2}, i+\frac{t}{2}+1\}$. We alter the watchmen's strategy depending on cases.

Case 1: At time $i+\frac{t}{2}+1$, w_1 does not move onto the arm containing s_2.

If $t' = i+\frac{t}{2}+1$, then w will be on v at time $i+\frac{t}{2}$, and if $t' = i+\frac{t}{2}$, then w will be on v at time $i+\frac{t}{2}-1$. We alter the watchmen's strategy so that w_1 stays on s_1 from time i to time $i+t$ and then takes over w's route for times greater than $i+t$. We adjust the route for w so that w takes over w_1's route for times greater than or equal to $i+\frac{t}{2}$. As w_1 does not move onto the stem containing s_2 at time $i+\frac{t}{2}+1$, the value of t^* has been increased.

Case 2: $t' = i+\frac{t}{2}$.

As in the previous case, if we alter the watchmen's strategy so that w_1 stays on s_1 from time i to time $i+t$ and takes over w's route for times greater than $i+t$. We also adjust the route for w so that w takes over w_1's route for times greater than or equal to $i+\frac{t}{2}$. Such an alteration is possible since w was originally on v at time $i+\frac{t}{2}-1$. The value of t^* is increased.

Case 3: $t' = i+\frac{t}{2}+1$ and w_1 moves onto the arm containing s_2 at time $i+\frac{t}{2}+1$.

By the minimality of t^* and the assumption no arm contains more than one guard at any given time and since s_2 must be visited every $t+1$ time units, s_2 must have been visited by w_2 at some time $t'' \in \{i-1, i, i+1\}$ such that at time $t''+1$, w_2 is no longer on s_2. It follows that at time $i+\frac{t}{2}$, w_2 is on either u, v or the vertex adjacent to v on the arm containing s_2. In this case, alter the watchmen's strategy as follows: w_1 stays on s_1 from time i to time $i+t$ and then takes over w's route for times greater than $i+t$, w_2 stays on s_2 from time t'' to time $i+t$ and then takes over w_1's route for times greater than $i+t$, and w takes over w_2's route at time $i+\frac{t}{2}$ or time $i+\frac{t}{2}+1$, contingent on three cases. If w_2's route in the initial strategy is on v at time $i+\frac{t}{2}$ then w (who is also on v at this time in the initial strategy) takes over w_2's route for times greater than $i+\frac{t}{2}$. If w_2's route in the initial strategy is on the vertex adjacent to v on the stem containing s_2 at time $i+\frac{t}{2}$ then w, remains on v at time $i+\frac{t}{2}$ and takes over w_2's route for times greater than or equal to $i+\frac{t}{2}+1$. Finally if w_2's route in the initial strategy is on u at time $i+\frac{t}{2}$, then w (who must be on u or v at time $i+\frac{t}{2}-1$, as w is on v at time $i+\frac{t}{2}$) takes over w_2's route for times greater than or equal to $i+\frac{t}{2}$. In each case, the value of t^* is increased.

To summarize: we have shown that given any strategy $\mathcal{S}$ which t-monitors the leaves of G, there is another strategy $\mathcal{S}'$ which uses the same number of guards, t-monitors the leaves of G, and has two watchmen w_1 and w_2 who are the only guards which visit s_1 and s_2. As s_1 and s_2 need to be visited every $t+1$ time units, w_1 and w_2 only move on vertices of F and the vertex u. Clearly, $G[V(G)-V(F)] \in \mathcal{G}_t$ and hence by the induction hypothesis, exactly $\left\lfloor \frac{2(n-|V(F)|)+t-2}{t+3} \right\rfloor$ watchmen are required to watch all the leaves of $G[V(G) - V(F)]$. Suppose u is not a stem or a leaf in $G[V(G) - V(F)]$. In this case, w_1 and w_2, whose movements are restricted to F and the vertex u, can not protect any leaves in $V(G) - V(F)$. Hence G requires

$$\left\lfloor \frac{2\,(n - |V(F)|) + t - 2}{t+3} \right\rfloor + 2 = \left\lfloor \frac{2n + t - 2}{t+3} \right\rfloor$$

guards to t-monitor its leaves.

On the other hand suppose u is a stem or a leaf in in $G[V(G) - V(F)]$. Note that this implies $n \equiv 1, 2, \frac{t+6}{2} \pmod{t+3}$. Let u' be the leaf adjacent to u in G if u is a stem and let $u' = u$ if u is a leaf in $G[V(G) - V(F)]$. Set $F' = F \cup \{u'\}$ if $n \equiv 1, \frac{t+6}{2} \pmod{t+3}$ and set $F' = F \cup \{u', u\}$ in the case $n \equiv 2 \pmod{t+3}$. Clearly, $G[V(G) - V(F')] \in \mathcal{G}_t$ and hence by the induction hypothesis, exactly $\left\lfloor \frac{2(n-|V(F')|)+t-2}{t+3} \right\rfloor$ watchmen are required to watch all the leaves of $G[V(G) - V(F')]$. Therefore at least

$$\left\lfloor \frac{2\,(n - |V(F')|) + t - 2}{t+3} \right\rfloor + 2 \geq \left\lfloor \frac{2\,(n - (t+5)) + t - 2}{t+3} \right\rfloor + 2 = \left\lfloor \frac{2n + t - 6}{t+3} \right\rfloor$$

guards are required to t-monitor the leaves of G. The result follows from the fact that $t \geq 4$ and $n \equiv 1, 2, \frac{t+6}{2} \pmod{t+3}$. ■

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