AUSTRALASIAN JOURNAL OF COMBINATORICS
Volume **44** (2009), Pages 287–296

Avoiding rainbow induced subgraphs in edge-colorings*

Chelsea Sackett   Maria Axenovich

Department of Mathematics
Iowa State University
Ames, IA 50011
U.S.A.
chelsea@iastate.edu   axenovic@iastate.edu

Abstract

Let H be a fixed graph on k edges. For an edge-coloring c of H, we say that H is rainbow, or totally multicolored if c assigns distinct colors to all edges of H. We show, that it is easy to avoid rainbow induced graphs H. Specifically, we prove that for any graph H (with some notable exceptions), and for any graph G, $G \neq H$, there is an edge-coloring of G with k colors which contains no induced rainbow subgraph isomorphic to H. This demonstrates that, in a sense, induced subgraphs do not have "anti-Ramsey"-type properties.

1 Introduction

Let $G = (V, E)$ be a graph. Let $c : E(G) \to [k]$ be an edge-coloring of G. We say that G is *monochromatic* under c if all edges have the same color, and we say that G is *rainbow* or *totally multicolored* if all edges of G have distinct colors. A graph $G' = (V', E')$ is an induced subgraph of G if $V' \subseteq V$ and $e \in E'$ if and only if $e \in E$. Ramsey, see [12], has shown that the monochromatic subgraphs are unavoidable in colorings of large complete graphs with fixed numbers of colors. Erdős, Simonovits and Sós, see [7], proved that the totally multicolored subgraphs are unavoidable if the number of colors used on the graphs is large enough. When considered colored subgraphs are induced, the situation becomes more subtle. Deuber, see [3], proved the induced variant of Ramsey theorem by showing that for any two graphs H_1 and H_2, there is a graph G such that in any coloring of edges of G in two colors 1 and 2, there is an induced subgraph of G isomorphic to H_i which is monochromatic of color i. This demonstrated that monochromatic induced subgraphs are also unavoidable

* This research was sponcored in part by Iowa State University Freshman Honors program.

in any coloring of some graphs. The induced Ramsey number $r_{\text{ind}}(H_1, H_2)$ is the smallest order of such a graph G. The bounds on this number were first established by Deuber, Erdős, Hajnal and Pósa, also by Rödl, see [3, 6, 12, 8]. But the best bounds up to date are due to Kohayakawa, Prömel and Rödl, see [10], giving that $r_{\text{ind}}(H_1, H_2) \leq |H_2|^{c|H_1|\log\chi(H_2)}$, for $|V(H_1)| \leq |V(H_2)|$ and an absolute constant c. In this paper, we show that the anti-Ramsey result does not extend to induced subgraphs. Specifically, we show that any graph H, with a few exceptions, can be avoided as a rainbow induced subgraph in edge-colorings of any graph.

Definition 1 *Let H be a graph with k edges and no isolated vertices. For a graph G, we say that G arrows H and write*

$$G \to H$$

if and only if every possible edge-coloring of G with k colors contains an induced rainbow subgraph isomorphic to H. We write

$$G \nrightarrow H$$

if G does not arrow H.

Definition 2 $f(H) = \max\{n :$ *there is a graph G on n edges such that $G \to H$*$\}$.

We denote by K_n, S_n, P_n, C_n a complete graph, a star, a path, and a cycle on n vertices, respectively. Note that $S_2 = K_2 = P_2$, $P_3 = S_3$. A vertex-disjoint union of graphs G and H is denoted $G + H$; a graph which is a vertex-disjoint union of n copies of a graph G is denoted nG. We say that an acyclic graph H is a *double star* if and only if it is not a star and it has an edge e, called *central*, so that all edges in H excluding e are adjacent to e; we call the vertices of the central edge *central vertices*. A vertex-disjoint union of stars is *almost balanced* if either 1) it has at least two stars such that one star has m edges and all other stars have $m+1$ edges, $m \geq 1$ or 2) it has at least three stars such that one star has one edge, one star has m edges, and all other stars have $m+1$ edges, $m \geq 2$. We denote the set of all almost balanced unions of stars excluding $S_3 + S_2$ by Bal^*. Formally,

$$Bal^* = \{S_m + tS_{m+1} : m \geq 2, t \geq 1\} \cup \{S_2 + S_m + tS_{m+1} : m \geq 3, t \geq 1\} \setminus \{S_3 + S_2\}.$$

A double star is *almost balanced* if the number of edges incident to one vertex of the central edge is a and the number of edges incident to the other vertex of the central edge is $a+1$, for some $a \geq 2$. We denote the set of all almost balanced double stars by $Doub^*$.

Let

$$\mathcal{F}_\infty = \{S_n,\ nK_2,\ S_n + K_2 : \quad n \geq 2\},$$
$$\mathcal{F}_1 = Doub^* \ \cup\ Bal^*,$$
$$\mathcal{F}_2 = \{P_4\} \cup \{2S_2 + tS_3 : t \geq 1\};$$

see Figures 1, 2, 3 for illustration. Our main result is the following.

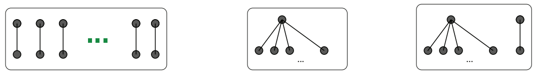

Figure 1: $\mathcal{F}_\infty$

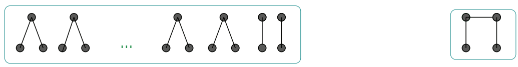

Figure 2: $\mathcal{F}_2$

Theorem 1 *Let H be a graph with k edges and no isolated vertices. Then*

$$f(H) = \begin{cases} \infty, & H \in \mathcal{F}_\infty, \\ k+2, & H \in \mathcal{F}_2, \\ k+1, & H \in \mathcal{F}_1, \\ k, & \textit{otherwise.} \end{cases}$$

Corollary 2 *Let H be a graph with no isolated vertices and such that $H \notin \mathcal{F}_1 \cup \mathcal{F}_2 \cup \mathcal{F}_\infty$. Let G be any graph not isomorphic to H. Then $G \not\twoheadrightarrow H$.*

Note that a similar function $f(H)$ can be defined for vertex-colorings. The corresponding problem is solved in [1], [2].

We prove the main theorem in the section 3. In section 2, we give a structural argument restricting our attention only to graphs H which are special forests, then treating the case when $f(H) = k$, $f(H) = k+1$, $f(H) = k+2$, and, finally, $f(H) = \infty$.

2 Lemmas and their proofs

If a graph G contains an induced subgraph isomorphic to H, we shall write $H \subseteq G$. For a subset E' of edges of a graph G, we shall denote $G[E']$ the induced subgraph of G spanned by the endpoints of edges in E', in other words, the smallest induced subgraph of G containing all edges from E'. In the following lemmas we shall assume that $|E(H)| = k$ and that H has no isolated vertices.

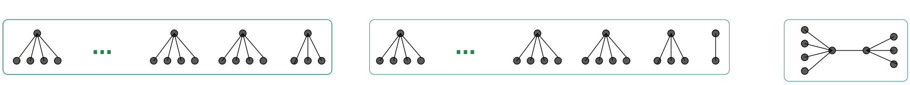

Figure 3: $\mathcal{F}_1$

Lemma 1 *Let H have k edges and let G have a set E' of t edges, $t < k$, such that $G[E']$ is not an induced subgraph of H. Then $G \not\to H$.*

Proof: Color edges from E' in t distinct colors and color the rest of $E(G)$ with the remaining $k - t$ colors. Then any rainbow subgraph of G on k edges must contain E'. However, $G[E']$ is not an induced subgraph of H. Thus, there is no rainbow induced copy of H in this coloring. □

Corollary 3 *Let H have k edges and n vertices. If G has a set of at most $k - 1$ edges on more than n vertices then $G \not\to H$.*

Lemma 2 *If H has k edges and two edges $e = \{u, u'\}$ and $e' = \{v, v'\}$ (note: u might be equal to v) such that $H - \{e, e'\}$ has no isolated vertices, then $f(H) = k$.*

Proof: Assume that $G \to H$ and $|E(G)| \geq k+1$. Consider a set of vertices $S \subseteq V(G)$ such that S induces H (such a set of vertices exists, otherwise by definition $G \not\twoheadrightarrow H$). Since $|E(G)| > |E(H)|$ and S induces H, there is an edge $e'' = \{x, y\} \in E(G)$ such that at least one of its endpoints is not in S. We will say, without loss of generality, that $x \notin S$. Let G' be a subgraph of G with edge set $E(G') = (E(H) \setminus \{e, e'\}) \cup \{e''\}$. Then $|V(G')| > |V(H)|$. By Corollary 3, $G \not\twoheadrightarrow H$. □

Lemma 3 *If $G \to H$, $H \notin \mathcal{F}_\infty \cup \{P_4\}$ and $|E(G)| > k$, then both H and G are a disjoint union of stars and at most one double star. Moreover, if H is the union of t stars and s ($s \leq 1$) double stars then G is also the union of t stars and s double stars.*

Proof: Claim 1 H is the disjoint union of stars and at most one double star.
Observe that Lemma 2 implies that H does not have any subgraph isomorphic to C_n, with $n \geq 4$. If G has a triangle, color its edges with colors $k, k-1$ and color the rest of the graphs with colors $1, 2, \ldots, k-2$; this coloring does not contain any induced rainbow subgraph on k edges. Since G has no triangles, H also has no triangles. So, we can assume that each component of H is a tree. Using Lemma 2, it is easy to observe that the H is disjoint union of stars and at most one double-star.

Claim 2 The number of components of G is at most the number of components of H.

Let H have t components. Since $H \notin \mathcal{F}_\infty$, H is not a matching, thus $t \leq k - 1$. Assume that G has at least $t + 1$ components. If $t < k - 1$, color at least $t + 1$ edges from distinct component of G with different colors and color the rest of the graph with the remaining $k - (t + 1)$ colors arbitrarily. If $t = k - 1$, color each component of G monochromatically. In both cases, any rainbow k-edge subgraph has at least $t + 1$ components, a contradiction.

Claim 3 G is the union of stars and at most one double star. Moreover, if H has no double stars, then G also has no double stars.

We may assume that $k \geq 4$ since otherwise $H \in \mathcal{F}_\infty \cup \{P_4\}$. Assume that G has a path of length 4, then take first, second and fourth edges of this path. These three edges induce a graph containing P_5. Since P_5 is not a subgraph of H, using Lemma 1, we arrive at a contradiction. Thus $P_5 \not\subseteq G$. This implies that $C_n \not\subseteq G$ for $n \geq 5$. We also have that $C_4 \not\subseteq G$ since three of its edges induce a graph which is not a subgraph of H. $C_3 \not\subseteq G$ since $k \geq 4$ and $C_3 \not\subseteq H$. This implies that G is a disjoint union of stars and double stars. Assume that G contain two double stars.

Let $k \geq 5$. Consider two nonadjacent edges e, e' from one double star and two nonadjacent edges e'', e''' from another double star. Then e, e', e'', e''' induce $P_4 + P_4$ in G. Since $P_4 + P_4 \not\subseteq H$, we arrive at a contradiction with Lemma 1.

Let $k = 4$. We see that H must contain a double star since there are two edges in G inducing a double star. Let us color the first double star of G with colors 1 and 2 and color the second double star with colors 3 and 4. Then no rainbow subgraph of G has a double star, a contradiction. Therefore, G must be the disjoint union of stars and at most one double star.

Assume now that H has no double stars and G has a double star. Pick two nonadjacent edges from a double star of G to arrive at a contradiction with Lemma 1. This concludes the proof of Claim 3.

Claim 4 The number of components of G is at least the number of components of H.

If H is a disjoint union of t stars, the Claim follows immediately since H is an induced subgraph of G and since G is the disjoint union of stars itself. If H is the disjoint union of t stars and a double star, by Claims 1 and 3, G is the disjoint union of at most t stars and a double star. If the number of stars in G is less than in H then any induced subgraph of G containing a double star has less than t stars, a contradiction. If H is a double star then since G contains a double star the Claim follows. This concludes the proof of the Lemma. □

Lemma 4 *If H is the union of stars and at most one double star and $H \notin \mathcal{F}_\infty \cup \mathcal{F}_1 \cup \mathcal{F}_2$ then $f(H) = k$.*

Proof: Let $G \to H$ and G has at least $k+1$ edges, $H \notin \mathcal{F}_1 \cup \mathcal{F}_2 \cup \mathcal{F}_\infty$. By Lemma 3, we see that H is 1) a union of stars, 2) a double star, or 3) a union of stars and exactly one double star.

1. Let H be the disjoint union of t stars, $t \geq 2$. Note that $H \neq S_k + S_2$. Then G is also the disjoint union of t stars by Lemma 3. Let $m(G)$ and $m(H)$ be the sizes of the largest stars in G and H, respectively. Let $m'(G)$ and $m'(H)$ be the second largest sizes of stars in G and H, respectively, with $m'(G) < m(G)$ and $m'(H) < m(H)$. We have that $2 \leq m(H) \leq k-2$, since otherwise H is either a star or a disjoint union of a star and a single edge, or a matching. Since $H \subseteq G$, $m(G) \geq m(H)$. If

$m(G) > m(H)$, then color $m(H)+1 \leq k-1$ edges of the largest star in G distinctly, and use the rest of the colors for the rest of the edges of G. Then any rainbow induced subgraph in G with k edges will have a star that is larger than the largest star in H, so it cannot be isomorphic to H. Therefore, we have that $m(G) = m(H) = m$. Assume first that H has only one star of size m.

Case 1: G has only one star of size m.
Color the edges of G such that the largest star has $m-1$ colors. This is always possible to do since $|E(G)| > |E(H)|$. Then any rainbow subgraph of G has a largest star of size at most $m-1$, a contradiction.

Case 2: G has at least two stars of size m, and $m'(H) \leq m-2$.
Color one of the largest stars of G with $m-1$ colors and color the rest of the graph with other remaining colors. Then any rainbow subgraph of G on k edges will have a star of size $m-1$, a contradiction.

Case 3: G has at least two stars of size m and $m'(H) = m-1$.
Since $H \notin \mathcal{F}_1 \cup \mathcal{F}_2 \cup \mathcal{F}_\infty$, we have that H has at least three components and that there are at least two edges not contained in the two largest stars, (otherwise $H = S_{m+1} + S_m + S_2$), that is, $2m-1 \leq k-2$, and $2m \leq k-1$. Multi-color two largest stars in G using $2m$ colors. Then use the remaining colors to color the rest of the graph arbitrarily. Then any rainbow induced subgraph of G on k edges will contain at least 2 stars of size m, a contradiction.

Now, let H have $t \geq 2$ stars of size m. Then G has at least t stars of size m. Let H' be obtained from H by deleting $t-1$ largest stars and G' is obtained from G by deleting $t-1$ largest stars. Then we see from above that there is a coloring c' of G' with $|E(H')|$ which does not contain a rainbow copy of H'. Let us multicolor $t-1$ largest stars of G and use coloring c' with new set of colors on the remaining graph (isomorphic to G'). Then clearly, there is no rainbow copy of H produced by this coloring, a contradiction to the fact that $G \to H$.

2. Let H be a double star. Recall that H is not almost balanced. Then G is a double star by Lemma 3. Let $a+1$ and $b+1$ be the number of edges incident to each vertex of the central edge of H, with $a \geq b \geq 1$, and let $c+1$ and $d+1$ be the number of edges incident to vertices v, v' of the central edge of G, respectively, with $c \geq d \geq 1$. Observe that $d \geq b$, $c \geq a$, otherwise $H \not\subseteq G$. If $c > a$, color $a+1$ edges incident to v with distinct colors, and color the rest of the graph arbitrarily with new colors. Then any rainbow subgraph of G on k edges will have maximum degree greater than a, thus this graph is not isomorphic to H.

If $c = a$, color all $c+1$ edges of G incident to v with c colors, color the rest of the graph arbitrarily. Since H is not almost balanced, this coloring does not contain a rainbow copy of H. Thus $G \not\to H$.

3. Let H be a disjoint union of at least one star and exactly one double star.

Let the double star of H have s edges, the stars of H have $k-s$ edges. Lemma 3 implies that G is a union of stars on at least $k-s$ edges and a single double star on at least s edges.

Case 1: The number of edges in the stars of G is greater than $k-s$.
Color the double-star of G with $s-1$ colors and color the rest of G with remaining $k-(s-1)$ colors. Then any rainbow subgraph has either no double star or a double star on at most $s-1$ edges, a contradiction.

Case 2: The number of edges in the stars of G is $k-s$. This means that the double star of G has at least $s+1$ edges. If $k-s \geq 2$, color the double star in G with $s+1$ colors and color the stars in G with $k-s-1$ new colors. Thus, there is no rainbow H in this coloring of G. If $k-s=1$ then color the central edge of G and an isolated edge with the same color and color the rest of the edges arbitrarily with new colors. This coloring does not have a rainbow copy of H either. Thus, $G \not\to H$. This contradiction concludes the proof of the Lemma. □

Lemma 5 *If $H \in \mathcal{F}_1$ then $f(H)=k+1$.*

Proof: Let $|E(H)|=k$. Assume first that H is an almost balanced union of n stars, $n \geq 2$, with one star having $m-1$ edges and all others having m edges, $m \geq 2$. Any k-coloring of $G=nS_{m+1}$ has a rainbow induced subgraph isomorphic to H, so $G \to H$. If $|E(G)|>k+1$, and $G \to H$, by Lemma 3, G is the disjoint union of the same number of stars as H. Observe first that $m+1 \leq k-1$, since otherwise $m=k-1$ and $H=S_3+S_2 \in \mathcal{F}_\infty$. There is at least one star in G with at least $m+1$ edges. By coloring this star with $m+1$ distinct colors and the rest with the remaining colors, there will be an S_{m+2} in any rainbow induced subgraph of G with k edges, and thus $G \nrightarrow H$. Therefore, $f(H)=k+1$.

Assume now that H is an almost balanced union of stars with one star having one edge, one star having $m-1$ edges, and all others having m edges, $m \geq 3$. Consider the graph $G=nS_{m+1}+K_2$. Any k-coloring of G has a rainbow induced subgraph of H, so $G \to H$. If $|E(G)|>k+1$, and $G \to H$, by Lemma 3, G is the disjoint union of the same number of stars as H. Then if there is a star with at least $m+1$ edges, color this star with $m+1$ distinct colors and the rest of the graph with the remaining colors. Then there will be a S_{m+1} in any rainbow induced subgraph of G with k edges, and thus $G \nrightarrow H$. If there is no star on at least $m+1$ edges, then there is no star on only one edge. Then color each star in G with at least 2 different colors, so that any rainbow induced subgraph of G will have n stars each on at least 2 edges and cannot contain a copy of H. Note that this coloring does not exists if $m=2$. Therefore, $G \nrightarrow H$, and $f(H)=k+1$.

Now assume that H is an almost balanced double star having m edges adjacent to one central vertex and $m-1$ edges adjacent to the other. Consider the graph G which is a double star with m edges adjacent to each central vertex. Then any k-coloring of G will contain a rainbow induced H, so $f(H) \geq k$. But, if $|E(G)|>k+1$ and $G \to H$, again by Lemma 3 we have that G is a double star. Then G has a vertex v of degree higher than the maximum degree of H. Color the central edge of G with color 1, color other edges incident to v with as many distinct colors as possible, color the rest of the graph with the remaining colors if there are any or with color 1. Thus $G \nrightarrow H$. So, $f(H)=k+1$. □

Lemma 6 *If* $H \in \{P_4,\ 2S_2 + tS_3 :\ t \geq 1\}$ *then* $f(H) = k + 2$.

Proof: Let $H = P_4$, $G \to P_4$, and $|E(G)| \geq 6$. Then Lemma 1 implies that G has no subgraphs isomorphic to K_3 or C_4 and has no induced subgraphs isomorphic to $2K_2$. This implies that G has no induced subgraph isomorphic to C_n with $n \geq 6$, $n = 3$, or $n = 4$ and no induced subgraph isomorphic to P_5. We see also that G must be connected. Assume that G has an induced subgraph C isomorphic to C_5. Then, since G has at least 6 edges and G is connected, we see that there is an edge $e = \{x, y\}$ incident to some vertex of C, i.e., $x \in V(C)$, $y \notin V(C)$. Since G has no C_3 or C_4, we see that y is not adjacent to any vertex of C thus G has an induced P_5, a contradiction. Thus, G has no cycles, so it is a tree with no induced subgraph isomorphic to P_n with $n \geq 5$. Since $P_4 \subseteq G$, G is a double star. Let the two vertices of the central edge in G be v and v'. Assume, without loss of generality that v has degree at least 3. Color two non-central edges incident to v with distinct colors, and color all other edges the remaining color. Then any rainbow induced subgraph of G on three edges will be a star and will not be isomorphic to P_4. Therefore, $G \not\to H$. Thus, if $G \to H$, then $|E(G)| \leq 5$. On the other hand, it is easy to see that $C_5 \to P_4$. Thus $f(P_4) = 5$.

Let $H = 2S_2 + tS_3$, $t \geq 1$. Let $G \to H$ and $|E(G)| \geq k + 3$. By Lemma 3, we see that G is a union of $t + 2$ stars. Clearly, at least one of these stars has at least three edges. Color these three edges with distinct colors and color the rest arbitrarily with remaining colors. This coloring does not contain rainbow H, a contradiction. On the other hand, let $G = (t + 2)S_3$. It is easy to check that $G \to H$. Thus $f(H) = k + 2$. □

Lemma 7 *Let* $H \in \mathcal{F}_\infty$. *Then* $f(H) = \infty$.

Proof: Recall that $\mathcal{F}_\infty = \{S_n, nK_2, S_n + K_2 : n \geq 2\}$.

Let $H = S_n$, with $n = k + 1$. Observe that $S_m \to S_n$ for $m \geq n$ since any n edges of S_m form an induced S_n, and therefore any k-edge coloring of S_m will contain a rainbow induced S_n. Since m can be arbitrarily large, $f(S_n) = \infty$.

Let $H = kK_2$ with $k \in \mathbb{N}$. Consider $G = mK_2$ with $m \geq k$. Then by coloring G in k colors, we can choose any k edges that have distinct colors, and these edges will form an induced subgraph of H. Therefore, $G \to H$ for all $m \geq k$, so $f(H) = \infty$.

Let $H = S_k + K_2$. Consider the graph $G = S_m + K_2$, $m \geq k$. Consider a coloring of $E(G)$ in k colors. Assume that an independent edge has color 1. Then there is a star in G using colors $2, 3, \ldots, k$. Therefore, $G \to H$. Since m can be arbitrarily large, $f(H) = \infty$. □

3 Proof of the main theorem

Let H be a graph on k edges. If $H = P_4 \cup \{2S_2 + tS_3 : t \geq 1\}$ then Lemma 6 implies that $f(H) = k + 2$. If $H \in \mathcal{F}_\infty$ then Lemma 7 claims that $f(H) = \infty$. Thus we can assume that $H \notin \mathcal{F}_\infty \cup \mathcal{F}_2$.

Let $G \to H$ and let G have more than k edges. Lemma 3 implies that for some integers $t \geq 1$ and $s \in \{0, 1\}$, both G and H are vertex disjoint unions of t stars and s double stars. This is used to prove Lemmas 4 and 5. If $H \notin \mathcal{F}_1$ Lemma 4 implies that $f(H) = k$. If $H \in \mathcal{F}_1$ then Lemma 5 implies that $f(H) = k + 1$. This concludes the proof of the main theorem.

References

[1] M. Axenovich and R. Martin, Avoiding induced rainbow subgraphs in vertex-colorings, *Electron. J. Combin.* 15 (2008), no. 1, Research paper 12, 23 pp.

[2] M. Axenovich, On subgraphs induced by transversals in vertex-partitions of graphs, *Electron. J. Combin.* 13 (2006), no. 1, Research paper 36, 11 pp.

[3] W.A. Deuber, Generalizations of Ramsey's Theorem, in *Infinite and finite sets*, (Colloq., Keszthely, 1973; dedicated to P. Erdős on his 60th birthday), Vol. I, 323–332. Colloq. Math. Soc. Janos Bolyai 10, North-Holland, Amsterdam, 1975.

[4] W.A. Deuber, *Canonization*, Combinatorics, Paul Erdős is eighty, Vol. 1, 107–123, Bolyai Soc. Math. Stud., Janos Bolyai Math. Soc., Budapest, 1993.

[5] N. Eaton and V. Rödl, A canonical Ramsey theorem, *Random Structures Algorithms* 3 (1992), no. 4, 427–444.

[6] P. Erdős, A. Hajnal and L. Pósa, Strong embeddings of graphs into colored graphs, in *Infinite and finite sets* (Colloq., Keszthely, 1973; dedicated to P. Erdős on his 60th birthday), Vol. I, 585–595. Colloq. Math. Soc. Janos Bolyai, 10, North-Holland, Amsterdam, 1975.

[7] P. Erdős, M. Simonovits and V.T. Sós, Anti-Ramsey theorems, in *Infinite and finite sets* (Colloq., Keszthely, 1973; dedicated to P. Erdős on his 60th birthday), Vol. II, 633–643. Colloq. Math. Soc. Janos Bolyai, Vol. 10, North-Holland, Amsterdam, 1975.

[8] I. Gorgol and T. Łuczak, On induced Ramsey numbers, Cycles and colourings (Stará Lesná, 1999). *Discrete Math.* 251 (2002), no. 1-3, 87–96.

[9] R. Graham, B. Rothschild and J. Spencer, *Ramsey theory.* Second edition. Wiley-Interscience Series in Discrete Mathematics and Optimization, New York, 1990. xii+196 pp.

[10] Y. Kohayakawa, H.J. Prömel and V. Rödl, Induced Ramsey numbers, *Combinatorica* 18 (1998), no. 3, 373–404.

[11] F.P. Ramsey, On a problem of formal logic, *Proc. London Math. Soc.* (2) 30 (1929), 264–286.

[12] V. Rödl, *Master's thesis*, 1973.

[13] D. West, *Introduction to Graph Theory,* Second Edition, Prentice Hall (2001), xx+588 pp.

(Received 19 May 2008; revised 6 Mar 2009)