Orthogonal layouts using the Deltahedron heuristic

J. W. Giffin
K. Watson
Department of Mathematics
Massey University
Private Bag 11222
Palmerston North
NEW ZEALAND

L R. Foulds
Department of Management Systems
University of Waikato
Private Bag 3105
Hamilton
NEW ZEALAND

ABSTRACT

The Deltahedron heuristic has been demonstrably effective for generating highly-weighted maximal planar adjacency graphs for use in facilities layout problems. In this paper, we present a linear-time algorithm for constructing an orthogonal block plan dual to such an adjacency graph which guarantees that all prescribed adjacencies are preserved, all facility area requirements are satisfied and facility shapes are at worst topologically equivalent to T-shapes.

1. Introduction and background

The graph theoretic approach to facility layout can be implemented in two parts: firstly, construct an adjacency graph, and, secondly, develop its orthogonal dual, frequently called a block plan, block layout or floorplan. The vertex-weighted edge-weighted adjacency graph specifies the pairwise adjacency of all facilities (rooms, machines) for a given problem instance and may be based on the consideration of maximization of relationship chart scores or on the minimization of overall transportation cost. We assume that all edge weights are non-negative. The associated vertex weights prescribe the area that each facility should have in the final layout. No facility shape requirements can be prespecified, but block plans whose facilities are as regularly shaped as possible are preferable.

Geometric duality requires that the adjacency graph be planar. In the optimization context, this translates to maximal planarity, since adding further edges of zero weight to

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a non-maximally planar adjacency graph cannot adversely affect the overall edge-weight sum. The maximal planarity assumption also provides structural regularity in an adjacency graph, which may be used to advantage in constructing its corresponding layout. Foulds (1983), Hammouche and Webster (1985), Hassan and Hogg (1987) and Kusiak and Heragu (1988) discuss the major aspects of the graph theoretic approach to layout in detail.

Several techniques exist for constructing adjacency graphs from relationship score or transportation cost data. Foulds and Robinson (1978, 1979) introduced the Deltahedron heuristic whereby a maximal planar adjacency graph is constructed by a sequence of "insertions" of a new vertex and three edges into a triangular face, starting with a complete graph on four vertices (K_4). Green and Al-Hakim (1985) echoed these ideas. Foulds and Giffin (1985, 1987) extended the basic relationship score approach to include transportation costs and the concept of near-adjacency of facilities. Leung (1992) further generalised the work of Foulds and Robinson (1978) in the form of a greedy constructive heuristic. Al-Hakim (1991) and Eggleton and Al-Hakim (1991) provided a unified approach to improvement schemes for increasing the total weight of previously constructed graphs. Boswell (1992) presented a face-augmentation heuristic which she proved could create any maximal planar graph using an appropriate edge-weighting function.

Once the adjacency graph has been obtained, a corresponding block layout must then be constructed. The layout must reflect the pairwise adjacency requirements of the adjacency graph, and the area specification of each of the facilities. This more difficult phase has been previously addressed in the graph theoretic context in Hassan and Hogg (1989), where the (geometric dual of the) adjacency graph is translated into a block layout defined on a grid pattern. They assume that the area of each facility comprises an integral number of unit squares and that area variation between facilities is relatively small in order that facility shape irregularity be reduced. Their approach appears to fail in cases where the adjacency graph contains a separating triangle; Al-Hakim (1992) addresses this difficulty and extends the approach to overcome it. Giffin et al (1994) present another remedy using a modified insertion order. Giffin et at (1986) described a rudimentary approach for the generation of block plans from a restricted class of adjacency graphs generated by the Deltahedron heuristic in which any facility is to be placed at most two walls from the building exterior. Rinsma et al (1990) provided a mechanism, based on properties of maximal outerplanar graphs, for generating orthogonal duals from any maximal planar adjacency graph, including arbitrary area specifications and allowing for the possibility of non-convex plan boundaries. This technique, although theoretically efficient, does not yet appear readily adaptable to computer implementation. Welgama et al (1994) present an alternative knowledge-based approach which does not guarantee to provide in the layout all adjacencies specified in its dual graph.
Rinsma (1988) showed that if the adjacency requirements are specified by a tree instead of a maximal planar graph, a rectangular block plan (i.e. in which each facility is a rectangle) with arbitrary facility areas may be constructively obtained in all cases. A concept related to tree adjacency is that of a design skeleton, mainly applicable to spine layouts and to layouts not specifying rectangular exteriors. Several papers discuss these and related ideas, for example Montreuil and Ratliff (1989) and Montreuil et al (1987). If

the adjacency graph is instead defined to be maximal outerplanar, the guarantee of rectangularity in general is lost if facility area requirements must be satisfied (Rinsma (1987)). Rinsma et al (1990) provided an example of the smallest maximal planar adjacency graph for which no block plan with each room rectangular or L-shaped exists, irrespective of prescribed facility areas.

Other work is based upon various network representations in which cardinal point orientations must be predefined, and "horizontal" and "vertical" wall direction problems solved. In general, the graphs considered for dualization are not maximal planar, because the output requirements for the problems investigated usually specify that the block plan be rectangular. See, for example, Roth et al (1982).

Bhasker and Sahni (1988) provided a linear-time rectangular dualization procedure for "properly triangulated" planar graphs in the context of boundary-oriented circuit design, while Bhasker and Sahni (1987) develop a linear time algorithm for determining whether or not a rectangular dual exists corresponding to an instance of such a graph. Lai and Leinwand (1990) give a characterisation of adjacency graphs for which a rectangular block layout exists, and show how constructing such a layout is equivalent to solving a bipartite matching problem. Note, however, that none of these techniques allow for predefinition of facility area.

2. Generating an orthogonal block plan

2.1 Generating the adjacency graph

The class of adjacency graphs for which the approach we present here is applicable are those generated by the Deltahedron heuristic and its variants (Robinson and Foulds (1978), Foulds and Giffin (1985, 1987, 1990)), which have been shown empirically to construct highly-weighted maximal planar adjacency graphs, despite possessing an asymptotically arbitrarily poor worst-case performance guarantee (Dyer et al (1985)). In particular, we extend the approach of Giffin et al (1986) so that the dimensioned dual layout of any adjacency graph generated by the Deltahedron heuristic may be constructed.

The basic version of the Deltahedron heuristic may be summarized as follows. For simplicity, we consider only the objective of maximizing relationship scores. Consider a layout problem with (n–1) facilities, and the exterior facility (denoted by vertex 1), each represented by a vertex of the maximal planar weighted adjacency graph to be constructed. The graph is built up one vertex and three edges at a time, starting with an initial triangulation of four vertices, K_4. Input requirements are the initial K_4 and the "insertion order" in which the vertices will be processed. If the relationship chart data is given in the form of a matrix $R = [r_{ij}]$, define

$$w(i) = \sum_{j=1}^{n} r_{ij}, \qquad i = 2, ..., n$$

and reindex the vertices so that $w(2) \geq w(3) \geq ... \geq w(n)$. $w(i)$ is a measure of the total adjacency desirability score for facility i. The vertices of K_4 are then taken to be {1, 2, 3, 4} and the vertex insertion order is 5, 6, ..., n. Each vertex 5, 6, ..., n is successively inserted into the face of the triangulation which results in the largest increase in edge weight, where the weight associated with edge ij is r_{ij}. That is, if vertex v is to be inserted next, all faces in the triangulation built up so far are examined. The face (x, y, z) with vertices x, y and z yielding the largest sum

$$r_{xv} + r_{yv} + r_{zv}$$

is identified, edges xv, yv and zv, vertex v and faces (x, y, v), (x, z, v) and (y, z, v) are added, and face (x, y, z) deleted. (This is called "inserting" vertex v in triangle (x, y, z)). This process continues until all vertices have been inserted, yielding a maximal planar adjacency graph whose construction required no planarity testing phases.

In this paper, we assume that the exterior facility (w.l.o.g. labelled 1) is one of the vertices of the initial K_4. If this is not the case it can be shown that a revised insertion order can always be constructed in $O(n^2)$ time which does include $1 \in K_4$ and which generates precisely the same adjacency graph (see Watson (1994)).

2.2 Constructing an orthogonal dual graph

The initial graph $G = K_4$ may be drawn as in Figure 1(a). A (dual orthogonal) block plan B(G) corresponding to G is shown in Figure 1(b), in which (recalling that vertex 1 refers to the exterior facility) the areas of facilities 2, 3, and 4 need not yet be considered explicitly, allowing the undimensioned representation of the layout to be used. Subsequent simple scaling of facility areas in the orthogonal dual graph will incorporate the actual area data. The wall intersections of B(G), labelled J_1, J_2, J_3 and J_4 in Figure 1(b), are called 3-joints (also sometimes referred to as dual points), whereas those denoted by j_1, j_2, j_3 and j_4 we term 2-joints. Each 3-joint corresponds to the

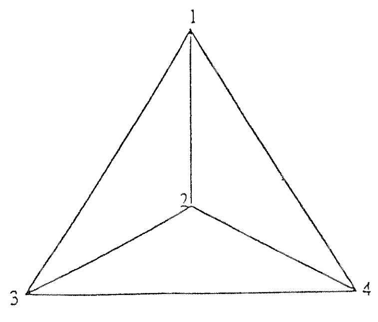

Figure 1(a) The initial adjacency graph, $G = K_4$

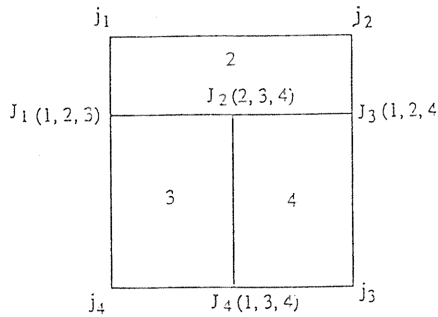

Figure 1(b) Block plan B(G)

confluence of the walls of three facilities (for instance, J_1 corresponds to facilities 1, 2, and 3, and J_2 to 2, 3, and 4), whereas 2-joints are merely a consequence of the orthogonalization process. Furthermore, following Foulds and Robinson (1978) and Green and Al-Hakim (1985), each 3-joint in B(G) is in one-to-one correspondence with a triangle in G (for example, J_1 with (1, 2, 3) and J_2 with (2, 3, 4)). It is convenient to directly implement this association in B(G) by labelling each 3-joint with its corresponding triangle in G, as depicted in Figure 1(b); this labelling may be easily updated to reflect any subsequent triangle addition or deletion in G. As long as this duality correspondence is maintained, explicitly constructing G becomes unnecessary for the construction of B(G) since both depict equivalent information, provided that the initial K_4 and the Deltahedron insertion order are known a priori.

If a triangle (f_1, f_2, f_3) in G is eliminated by the insertion of a new vertex, f, say, then, in B(G), the facility corresponding to f must be placed adjacent to those facilities corresponding to the vertices f_1, f_2 and f_3, in order that B(G) exactly reflect the new adjacencies in G. In order to maintain regularity of the shapes of facilities f_1, f_2 and f_3 in B(G) (and of facility f itself), an effective placement mechanism must be devised; a suitable choice involves placing f exclusively "inside" one of f_1, f_2 or f_3, bordering the other two facilities. (For simplicity, f_i will denote vertex f_i in any references to G, and the corresponding facility f_i in references to B(G)). The particular choice made must allow each facility in B(G) to be later "inflated" to reflect its actual area, irrespective of subsequent placements within it, whilst at the same time maintaining the required adjacencies (as specified by the edges of G) and maintaining the regular facility shapes that were defined during the placement process. Figure 2 shows the four possibilities for the insertion of facility 5 in the initial B(G) that retain the most regular shapes for the facilities. The 3-joints corresponding to the four original insertion triangles in G are indicated in B(G) by an asterisk. We describe such insertions as being **at** a particular 3-joint in B(G). Further, if a facility f_i is being placed within a facility f_j, we refer to f_j as the **placement host** of facility f_i.

The rationale behind the configuration in Figure 2(a) is as follows, and is typical of the insertion process at any stage of the construction of B(G). Facility 5 is being inserted at to 3-joint (1, 2, 3), i.e. adjacent to facilities 1, 2 and 3. To retain the rectangular boundary of the block plan, the placement host in B(G) will be either facility 2 or 3. If 2 were chosen as in Figure 3, B(G) would take in its most regular form one equivalent to Figure 3; all existing facilities (together with 5) would retain their regular (rectangular) shapes, and the required new adjacencies (15, 25, 35) would be assured, irrespective of the actual facility areas. However, if $a_4a_5 \geq a_2a_3$ (where a_i denotes the area of facility i) and the rectangular shape facility of each facility is maintained, adjacency 23 would be lost. Therefore, this form of placement will never be allowed; instead, the form of Figure 2(a) would be used, in which 3 is the placement host for 5. Areas a_4 and a_5 may, as the result of subsequent placement operations within them, be implicitly inflated by the areas of the facilities they then host, and this possibility is allowed for by ensuring that any subsequent placement is made consistently on the same "side" of a wall. A consequence of this requirement is that facility 2 (or whichever facility is placed at the “top” of the layout) will never be a placement host. Note that the relative facility areas here have been used only to motivate one aspect of the placement process; the final construction phase must proceed independently of such data. The resulting facility shapes should remain as regular as possible, and all required adjacencies in the plan must be provided for irrespective of variations in the areas of the facilities. As noted above some 3-joint descriptions require updating to reflect the insertion of vertex 5: face (1, 2, 3) is deleted

(a) 5 : (1, 2, 3)

(b) 5 : (1, 2, 4)

(c) 5 : (1, 3, 4)

(d) 5 : (2, 3, 4)

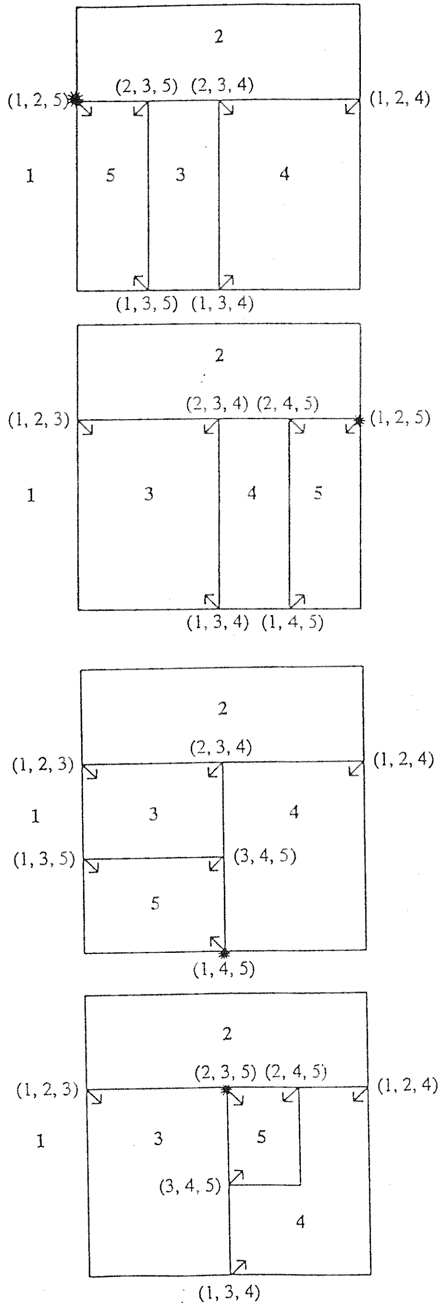

Figure 2 The four possibilities for placing facility 5 regularly into the initial B(G)

and replaced by faces (1, 2, 5), (1, 3, 5) and (2, 3, 5); 3-joint (1, 2, 5) therefore replaces (1, 2, 3), and the new 3-joints (1, 3, 5) and (2, 3, 5) are created, as shown in Figure 2(a). The placements of Figure 2(b) and 2(c) are similar to that of Figure 2(a), but placing 5 relative to (2, 3, 4) differs, since 5 must not be adjacent to 1. This cannot be accomplished whilst maintaining the rectangular shape of all facilities; either 3 or 4 will become L-shaped, depending on which is chosen as the placement host. The updated 3-joint descriptions, however, follow the previous pattern, giving the plan of Figure 2(d).

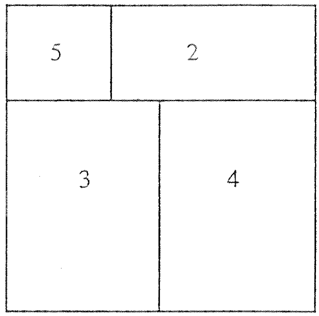

Figure 3 A rejected placement format

Before providing further details we illustrate the construction. Suppose that the insertion order and 3-joint (or insertion triangle) sequence for a given problem is:

5	:	(1, 2, 3)
6	:	(1, 3, 4)
7	:	(3, 4, 6)
8	:	(1, 2, 4)
9	:	(2, 3, 5)
10	:	(1, 3, 5)
11	:	(2, 3, 4)

A block plan B(G) corresponding to this data set is depicted in Figure 4, where the 3-joint descrip-tions have been suppressed. Note that, in terms of the placements outlined in Figure 2,

5	:	(1, 2, 3)	has the form of 2(a)
6	:	(1, 3, 4)	has the form of 2(a), rotated through 90° and reflected about a horizontal axis
7	:	(3, 4, 6)	has the form of 2(d)
8	:	(1, 2, 4)	has the form of 2(d), reflected about a vertical axis
9	:	(2, 3, 5)	has the form of 2(d), reflected about a vertical axis

10	:	(1, 3, 5)	has a form similar to that of 2(a)
11	:	(2, 3, 4)	has a form similar to that of 2(d)

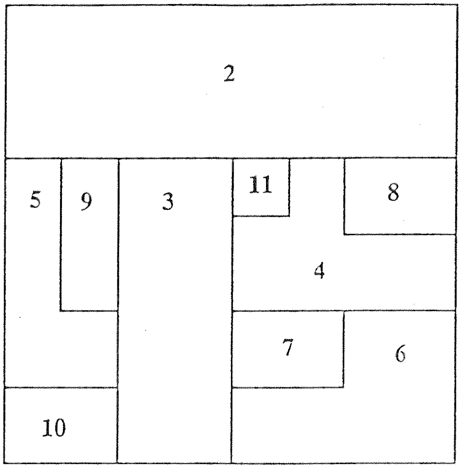

Figure 4 B(G) for the given insertion data

Clearly, the placement hosts have not been assigned arbitrarily; so-called placement directions (indicated by the arrows in Figure 2) specify these, and will be discussed later. Facility 7 has been placed within 6 rather than 4, this ensures (see later) that 4 will at worst become topologically equivalent to a T-shape. Facility 9 was placed within 5 rather than 3 because 5 contained a 2-joint at that stage, but 3 did not; shape regularity can be maintained (or worsened less) by placement within facilities possessing a 2-joint, if possible. Facility 10 was placed within 5 instead of 3, since, otherwise, the adjacency 35 could be lost if a_9 and a_{10} happened to be too large in relation to a_3 and a_5, as discussed above. We reiterate that knowledge of the actual facility areas is not required, in choosing the placement host.

Two "basic placement operations" are readily identifiable as those of Figures 2(a) and 2(d); each has the special variation in which the placement host is L-shaped rather than rectangular. It still has to be decided, however, which facility should be chosen as the placement host at each 3-joint of B(G) in order to apply these operations. To this end, call the placement operations of Figure 2(a) and 2(d) PO1 and PO2 respectively. The general form of PO1 is given in Figure 5(a), in which f_2 has been placed in f_1 at 3-joint J. For simplicity, the fact that f_1 is chosen as the placement host at J is indicated by a placement direction arrow emanating from J, the notation used in Figure 2. New 3-joints J_1 and J_2 are created by the placement of f_2. In the undimensioned dual layout, J_1 and J_2 are created by bisecting the wall connecting the adjacent joints. The placement direction at J_2 is as shown in order that any further facility placed relative to it impacts the least on the shape of f_2 and f_1; placement of f_3, say, in f_2, will leave f_1 and f_2 rectangular, whereas placement of f_3 in f_1 would leave f_2 rectangular but f_1 L-shaped.

(This is the same rationale as for PO1 itself, after appropriate reflection and rotation). Once the placement direction for J_2 has been established, the direction for J_1 must follow as indicated, in order to maintain the f_1f_2 adjacency irrespective of subsequent placements in either J_1 or J_2. When applying PO1 to an L-shaped placement host, the 3-joint J_1 bisects the wall connecting J and the 3-joint adjacent to J, with J_2 equidistant from the 2-joint, and orthogonality maintained .

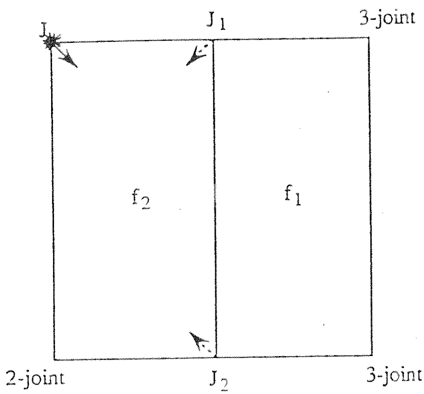

Figure 5(a) Placement Operation PO1

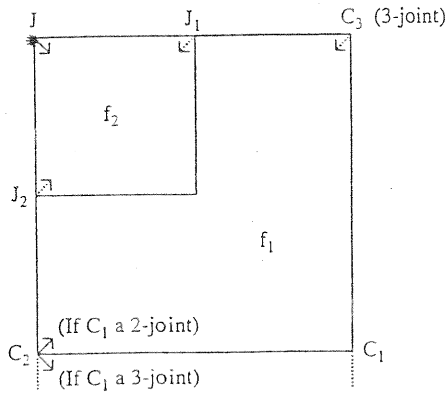

Figure 5(b) Placement operation PO2

The placement directions for J_1 and J_2 are defined analogously as for PO1, whereas those for the other existing 3-joints of f_1 have already been defined as a result of a previous application of PO2, which is considered next. The "nature" of 3-joint J in terms of its placement direction has not changed, so further placement at J, now within f_2, will take the same form as before the insertion of f_2. The general form of placement operation PO2 is given in Figure 5(b) for the case of a rectangular placement host. When the placement host is L-shaped, the obvious modifications are applied. Again the placement is of f_2 in f_1 at 3-joint J.

The two cases where joint C_1 is a 2-joint or a 3-joint may be considered together, as only the placement direction at 3-joint C_2 is affected in each case. Note that C_2 must be a 3-joint here, otherwise PO1 would be applied. The new 3-joints created are again labelled J_1 and J_2 and the reasons for choosing their placement directions are as follows. Had either been directed within f_1 instead of f_2, f_1 would prematurely lose its L-shape, as shown, for example, in Figure 6, in the case of inserting f_3 at J_1. Note in Figure 5(b), however, that C_1 must be a 3-joint; otherwise, f_3 would have been placed within f_1 using PO1. It is evident from their definitions that application of PO1 and PO2 will result in no prescribed adjacencies being lost, up to an (arbitrary) tolerance of minimum adjacency width. We also note that the insertion of only one further facility into the initial layout uniquely specifies all future placement directions.

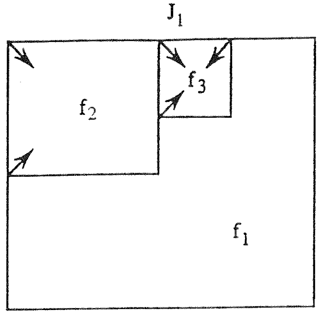

Figure 6 Incorrect insertion option following PO2

We reiterate that, in the undimensioned version of B(G), the locations of the 3-joints of a newly placed facility at 3-joint J are determined by simply halving the wall-length distance to the 3-joint adjacent to J (in the case of PO1) and to both of the 3-joints adjacent to J (in the case of PO2). It turns out that this may be done without compromising the actual required area of the facilities; the arbitrary areas in B(G) will assume their correct values in the subsequent "inflation" phase. Clearly, the amount of work required for the placement of each new facility is bounded above by a constant, so that the placement process is linear in the number of facilities.

In order to prove that we will indeed obtain a worst case room shape of T we will assume that the first insertion has been made already into the layout. Recall the four possibilities for this in Figures 2(a) to 2(d). We note that the placement directions as shown are by now completely specified. The insertion of the entering facility is made totally within the appropriate placement host, therefore at each insertion only the placement host can possibly have its room shape worsened. Note that any application of PO1 does not worsen the shape of f_1, therefore only PO2 can. Similarly if we perform PO2 when f_1 is a rectangle, then we easily see that f_1 becomes L-shaped. Hence the only way to create a T-shape is by performing PO2 on an L-shape placement host. Consider, therefore, PO2 with f_1 L-shaped. Referring again to Figure 5(b) the only way in which f_1 can become T-shaped is if C_1 is a 3-joint and we insert at C_3. (If C_1 is a 2-joint we are performing PO1). If we perform this placement, f_1 will become T-shaped, but, by construction, no placement directions will now lie within f_1. Hence f_1 can never again be a placement host and so cannot become any worse than T-shaped.

An outline of the procedure for constructing the undimensioned dual layout of a given maximal planar adjacency graph is given below in Algorithm Deltahedron_Dual.

Algorithm Deltahedron_Dual

Input: Deltahedron insertion order, DIO, in form (vertex, triangle), of maximal planar adjacency graph G; initial K_4 (*$1 \in K_4$*)

Output: an undimensioned orthogonal layout, dual to G.

```
begin
    Create initial B(G) configuration (*Figure 7*)
    (* reflections and rotations required for POi are implicit*)
    for each (vertex, triangle) of DIO do
    begin
        f2 = new facility (vertex) to be inserted
        J = 3-joint (triangle)
        f1 = placement host indicated by placement direction from J
        if    there is a 2-joint J' adjacent to J          then
        begin
            apply PO1 at J
            add placement directions for f2.
        end
        else  begin
            apply PO2 at J
            add placement directions for f2
        end
    end
end
```

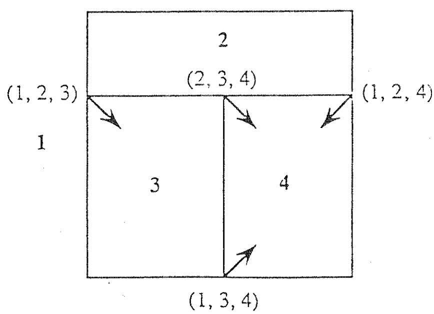

Figure 7 The initial B(G) configuration

Before describing the inflation phase, we provide a complete example of the orthogonal block plan construction process. Again suppose that the initial K_4 has vertices 1, 2, 3 and 4, that the insertion order is 5, 6, ..., 14, and that the insertion triangle (vertex) sequence is given as in Table 1. Included in Table 1 is the required placement operation for each insertion. In some cases, for instance the insertion of 7 in (1, 4, 6), operation PO1 is applied with rotation and reflection, but the basic principle is unchanged. The resulting undimensioned B(G) is given in Figure 8, and the corresponding adjacency graph is shown for completeness in Figure 9. Note that the arbitrary initial placement directions (defined in Figure 7) as within facility 4 had to be redirected within facility 3 following the insertion of facility 5; this is the only case where this can occur. Further placements in B(G) would take place from 3-joints in the directions indicated by the arrows in Figure 8.

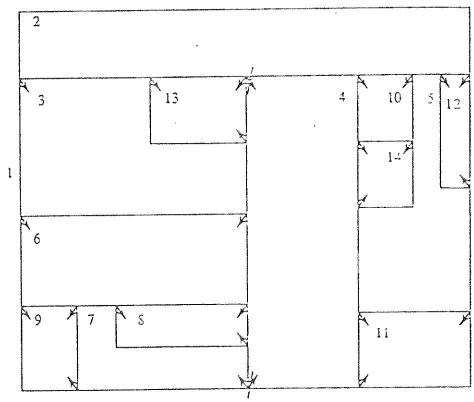

Figure 8 The completed undimensioned dual layout, B(G)

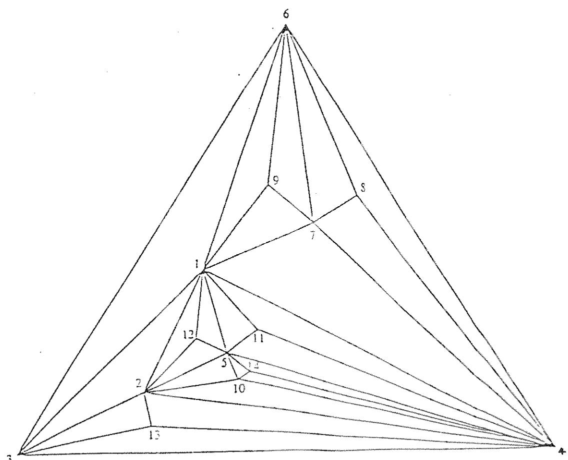

Figure 9 The adjacency graph for the example problem

Table 1 Data for the Example Problem

Facility	Triangle	Placement Operation
5	(1, 2, 4)	PO1
6	(1, 3, 4)	Reorient placement directions for (1, 3, 4), (2, 3, 4) since 2-joint adjacent at left of (1, 3, 4); PO1
7	(1, 4, 6)	PO1
8	(4, 6, 7)	PO2
9	(1, 6, 7)	PO1
10	(2, 4, 5)	PO2
11	(1, 4, 5)	PO1
12	(1, 2, 5)	PO2
13	(2, 3, 4)	PO2
14	(4, 5, 10)	PO1

2.3 Constructing the dimensioned layout by "inflation"

Inflating the undimensioned plan to reflect this data turns out to be straightforward, by sequentially considering any "blocks" of facilities created by either the initial B(G) configuration or by applications of placement operation PO1.

Firstly the exterior dimensions of the building need to be defined. The exterior envelope is assumed to be rectangular, and the enclosed layout area concomitant with $\sum_{i=2}^{n} a_i$ for an (n–1) facility problem. We motivate the ideas behind the general inflation procedure by example. Suppose that the areas (a_i) of facilities 2 through 14 are

$$[10, 6, 10, 9, 6, 7, 2, 27, 4, 2, 3, 2, 12]$$

so that the area of the final block plan is 100 units.

Referring to Figure 8, the horizontal interior wall of facility 2 is placed one-tenth of the vertical distance from the top of the layout, since $a_2 = 10$. The vertical interior walls of facility 4 may be placed according to the relative block areas,

$$a_3 + a_6 + a_7 + a_8 + a_9 + a_{13} : a_4 : a_5 + a_{10} + a_{11} + a_{12} + a_{14}.$$

Similarly, the horizontal interior walls of facilities 6 and 11 may now be placed according to

$$a_3 + a_{13} : a_6 : a_7 + a_8 + a_9$$

and

$$a_5 + a_{10} + a_{12} + a_{14} : a_{11}$$

respectively. The definition of the vertical interior wall for facility 9 then corresponds to the ratio

$$a_9 : a_7 + a_8.$$

The walls of facilities 8 and 13, which resulted from application of PO2, may be defined in the relative proportions $a_3 : a_{13}$ and $a_7 : a_8$. A sensible approach for these is to define the shape of 13 to be similar (in the geometric sense) to that of 3; likewise, the shape of 8 should be similar to that of 7.

Inflation after an instance of PO2 applied to an L-shaped placement host is slightly more complicated. The two rectangles containing facilities 10, 12, and 14, within facility 5 require this. In this case it is necessary to partition the block of area $a_5 + a_{10} + a_{12} + a_{14}$ so that the rectangle comprising 10 and 14 and the rectangle comprising 12 do not "overlap", i.e. the adjacency between 2 and 5 is retained. This can always be easily achieved.

Finally, the area containing facilities 10 and 14 may be partitioned in accordance with the ratio of a_{10} and a_{14}. Figure 10 shows the final scaled plan.

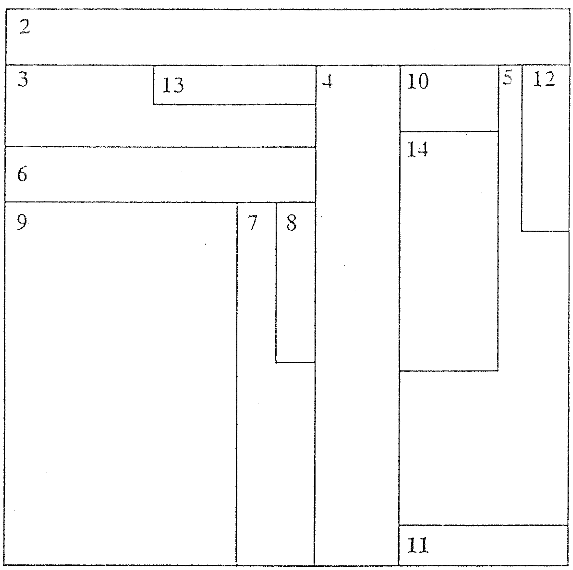

Figure 10 The final scaled block plan

The first step of the general inflation procedure is to determine the revised 3-joint positions for the facilities corresponding to the initial 3 vertices (other than 1) of K_4 (2, 3 and 4 in the example), by simple proportionality. From then on, the general principle is to find the revised 3-joint positions for all those facilities subsequently placed using the placement operations in priority order PO1, then PO2, (e.g. a PO2 cannot usually be performed before a PO1, except directly on the initial B(G)). Inflations arising from PO1 involve only simple rectangular proportionality conditions. PO2 requires the further use of similarity, or other simple schemes when placement hosts are L-shaped. The inflation procedure takes O(n) time, where n is the number of facilities, so that the complete orthogonal dualization scheme is also O(n).

3. Summary and conclusions

Given the insertion order of a maximal planar adjacency graph produced using the basic Deltahedron heuristic, we have shown how to construct its dimensioned dual orthogonal block plan, irrespective of the relative area specifications of the facilities, in time linear in the number of facilities. We note that it is not so straightforward to incorporate into the block plan any adjacency modifications caused by improvement phases developed for the Deltahedron heuristic (edge-interchange or vertex relocation). These would likely require extensive rearrangement of the plan using a sequence of new placement operations not possessing the simplicity of PO1 or PO2, or the necessity of restarting the layout phase from scratch.

If the dual of **any** given maximal planar adjacency graph is required, the techniques of Rinsma et al (1990) could be applied, with one exception: a check of the given adjacency graph should be made to test if a Deltahedron insertion order may be imposed upon it by successively deleting vertices of degree 3 from the graph; if this process may be continued until only K_4 is left, a reversal of the vertex deletion order yields the required insertion order.

If problem data is provided in the form of relationship score data or transportation cost data, available variations of the Deltahedron heuristic can provide an effective means of generating a highly-weighted adjacency graph and the required insertion order (see Foulds and Giffin (1985) and (1987)).

The rather rigid structure of the final (dimensioned) plan has some drawbacks, which are mitigated somewhat by the ease of its development. For instance, the facility placed at the "top" of the plan will end up a long and narrow "through room". Similarly, the other two rooms of the initial configuration will often end up extended and distended by the very nature of the placement process. Resorting to different definitions of K_4 (perhaps including the three largest facilities that should be adjacent to the exterior) or permitting a permutation of the positions of the first three facilities may result in an improved final layout.

Most of the difficulties outlined above are caused by the maximal planarity requirement. Its associated triangulation property may be used to advantage (only two placement operations are required), but fulfilling the concomitant adjacency requirements must necessarily result in "tentacle-like" corridors within some facilities. The incorporation of circulation zones in the form of communication paths (Gawad and Whitehead (1976)) or courtyards (Baybars (1982)) may reduce this difficulty. A certain amount of post-construction ornamentation, whereby some asthetic rearrangement of subgroups of facilities is undertaken, may reduce some shape distortion. In particular, for the example problem, facilities 10 and 14 could be placed within 4 instead of 5, given that the area of facility 11 is small enough to make any reshaping unnecessary. Such ornamentation would prove very difficult to automate, and firstly requires a rigorous definition of shape regularity.

Work is currently progressing on adapting the techniques in Hassan and Hogg (1989) and Rinsma et al (1990) to cater for arbitrary facility areas and to minimize shape distortion as much as is possible.

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